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THE MONIST

A LIBERAL SOCIALISTIC PROGRAMME

THE various socialistic systems previous to Marx were naughtily characterized by him as Utopian, because they were in substance but artificial constructions, which, while fully answering to this or that ideal of equity, did not take sufficient account either of the economic structure of the society that had to be reformed or of the ruling motives and respective forces of the social classes, thus neglecting just the two fundamental historic factors which dominate with iron determinism all social transformations. These Utopian socialistic schemes consequently claimed that the attractive form of equity and social well-being in which this or that reform was presented should have the inherent magic virtue of attracting to itself the whole body of society, which in an outburst of generous enthusiasm for this ideal regime would set it up right away.

To these Utopian systems of socialism Marx and Engels opposed their own system, which they modestly described as alone truly scientific, but which, while on one hand in complete contrast to the preceding ones, on the other does not differ substantially from them. For when once a given regime—that in which all production goes to the workers—is approved as just, Marx makes himself believe that he can get the facts to prove that the present regime, from the very fact of its becoming ever more unjust, tends toward the collectivist regime, which Marx considers the only one capable of realizing his ideal of equity. The thesis produces the antithesis from which the synthesis

will be formed in the Hegelian sense. It is not the case then that Marx examines the economic evolution objectively, without preconception or preference, so as to succeed in foreseeing whither it tends, but he first caresses the idea of a given regime, exactly like all the Utopian socialists who preceded him with the only difference that he thereafter, instead of presenting it ingenuously as fitted to convince and attract to itself all mankind united in brotherhood, attempts to force the facts to prove that this regime is the one toward which economic evolution tends irresistably and fatally, not by the will of men, but by the force of things.

At the same time the Marxian socialism is quite the reverse of the previous Utopian socialism just on account of this economic fatalism, which, by making the advent of the desired regime depend on the mechanical and fatal evolution of the economic process, entirely denies *a priori* the efficacy of law in general, and of the law of property in particular, as regards changing in one direction or in another the economic process itself.

The mischief done by this economic fatalism to the action of the socialist party, though not very gravely felt as long as this party was obliged, by the feebleness of its forces, to limit itself to a merely negative action of criticism, is severely felt now that the party, with its forces so notably increased as to put it in a position to act, would require a positive programme of action.

Now the only programme that Marxism has been able to propose is that of the violent revolutionary dispossession of that little group of magnates of capitalism to which Marx predicted the capitalist class would be reduced; and by this violent revolutionary act he had the illusive hope of setting up in one day the collectivist regime, which the inevitable concentration of manufacturing concerns, driven to the utmost limit, would without fail have already estab-

lished under the frail capitalistic involucre, and all in working order.

But neither the reduction of the possessors of capital to a small group of magnates nor the concentration *pari passu* of all the manufacturing concerns, driven to such an extreme limit as to set up of itself the collectivist regime in working order, has come to pass, and the Socialist party, unable to perform that one revolutionary act indicated to them by Marx, which at the present moment would mean smashing up the whole delicate mechanism of economic production, and at the same time without any other positive programme of action, has in all countries fairly lost its bearings. This want of any clear line of policy leaves it groping in the dark, a prey to fruitless and disorderly agitation, threatened with being broken up by continual new schisms into ever more numerous sections and subsections, incapable of combating the generally revived anarchy; and all this to the immense damage of the economic production of the country, which suffers from this chronic state of objectless revolutionary agitation and to the complete discredit of the Socialist party, which proves itself incapable of any reconstructive action.

The Socialist party will not be able to get out of this state of impotence in any other way than by absolutely renouncing the mechanical economic fatalism of the Marxian school and returning to the acknowledgement of the efficacy of law, and especially of the law of property, toward modifying the course of economic phenomena.

Be it well noted that we do not intend to go back to the ingenuousness of the Utopian Socialism which thought it enough to present a proposal that bore the mark of equity in order to obtain at once the unanimous assent of all social classes; but we maintain only that when once a given social class, hitherto of very little weight as a social factor,

increases in power so as to become a preponderating political factor it can, when risen to power or participation in power, modify the laws, and especially the law of property, by means of legislation, in accordance with its clearly understood and legitimate interest, and that these modifications of the law of property, to be imposed on the other classes more or less against their will, can effectively modify in its favor the course of the economic process.

Now this preponderance, as a political factor, of the wage-earning working classes over all the classes and subclasses of the *bourgeoisie* united, is a fact already accomplished, or soon to be so, in all countries. The moment has just come therefore for the Socialist party to return to that *legal socialism* which the Marxian fatalism has wrongly disdained too much, even though the various systems certainly erred, hampered as they were by too great respect for the rights of property beyond what is strictly necessary and sufficient to stimulate to the utmost to work and saving, through excessive timidity resulting in excessive slowness and meagreness of means to be used for the desired gradual nationalization of private property.

A system there is of accelerating, much more than could any other system hitherto devised, this process of nationalization, without throwing into disorder the existent organization of production, or injuring in any way those economic activities and that virtue of thrift which today are more than ever necessary for the increase of economic production. It is that of applying to the succession duties of the State, which is raised to the dignity of co-heir, the principle of progressiveness, not only as regards the extent of the patrimony or the degree of relationship, but also the *age* of the patrimony. That is to say the total amount of the patrimony left by the deceased would be divided into so many portions according to the number of times each portion had passed from hand to

hand, by way of succession or donation or dowry, before arriving at the deceased, and the rate of participation of the State in the inheritance of these various portions of the patrimony would increase with the increase of the number of transmissions undergone by them.

The State, for example, might continue to levy on the portion due to the labor and savings of the deceased not more than what it does today with the succession duties; but on the portion inherited by the deceased from his father and due to the labor and savings of the latter, the State would levy a much larger quota, for example 50 per cent; and on the portion that had come to the deceased, through his father, from his grandfather, the original accumulator of this portion, or, more generally, that had already undergone two transmissions whatsoever as private property, the State might inherit a much larger quota, reaching even 100 per cent (which would render the division of each patrimony into three parts sufficient, as nothing could be inherited from a great-grandfather). Owing to the impossibility, however, of searching out the most distant sources of the patrimonies now existing, and also for reasons of equity, all the patrimonies existent at the moment of the promulgation of the new law might be considered as due, for example, to the extent of a third part, to the work and savings of the present proprietor, and of two-thirds as inherited by one transmission as private property; and this as regards the State succession duty to be levied at the death of all proprietors now living.

It is evident that such a reform of the rights of succession would more effectively stimulate to saving than does the present right to bequeath all. In fact, as regards one's own children, every sum saved by the heir of a given patrimony would come to have in his eyes *a value much greater, even three or four or five times greater,* than an equal sum inherited by him; while today the heir of immense posses-

sions, which he knows he can bequeath in their entirety to his children, is in no way urged to that hard work, that severe abstinence, which would be necessary to him in order to double or triple the patrimony he has inherited.

The State, co-heir to the extent of 50 per cent in the quotas of patrimonies that have already undergone one transmission by way of succession or donation or dowry, and to the extent of 100 per cent in those that have undergone two transmissions, would levy the part belonging to it, not in money but in kind: land, buildings, State and Treasury bonds, shares and debentures in limited and other companies, etc., precisely like the other heirs.

State, treasury, provincial and communal bonds, as they would become the property of the State, should be immediately destroyed. This gradual amortisement of public debts would progressively free the States and Provinces and Communes from the enormous burden of the payment of interests, which is today a heavy handicap on all the really productive economic activities. This notable diminution of the burdens on the public bodies, along with the growing incomes which these bodies would derive from the rent of lands, dwelling houses, buildings in general, as they would become nationalized, would allow of the gradual passage from a system of finance on the basis of taxes to one on the basis of rents.

The principle of share-holding by the State, or better still by the great national trade unions of manufacturing workmen, would be put into effect, in a gradual way, as gradually the shares of limited and other companies were inherited by the State.

It will be seen that this proposal would satisfy the fundamental principle of socialism, that of greater equity, and would carry out its maxim of socialization of the instruments of production and of all capitals in general, which it justly considers necessary for the complete emancipation

of the workmen. The gradual character of the process of nationalization would permit of the passage from the present to the new regime by peaceful and legal means, without disorganizing the delicate mechanism of economic production, without requiring even the improvised creation of an entirely new bureaucratic organism; it would, on the contrary, render possible the gradual transformation of some of the organs already in existence so as to fit them for the new task, as this increases in extent and importance and would give the needed time for the process of economic production to adapt itself, by increased production of necessities and diminished production of luxuries, to the new and more equitable regime. As regards the coexistence of nationalized capital with always new private capital, the profits of this latter would represent in a continually diminishing measure the exploitation of the work of others on the part of heirs who have had no share in the accumulation of their possessions, and in ever increasing measure would represent the just reward of abstinence given to one who had created new capital effectively by his savings. Moreover, from the fact that the proportion of the total of private capital to the total of that nationalized would keep diminishing, especially through strong progressivenesses of the duty as that above indicated, the proportion also between the quota of the social annual income belonging as profits to private capital and the quota belonging to labor would equally keep decreasing, in favor of the latter.

The proposal would at the same time satisfy the fundamental principles of orthodox economics and of the free-contract regime, inasmuch as it would not assign to the State any coercive function in the way of the military organization of labor, as collectivism would do, but merely that of furnishing to the workmen, through suited self-governing organs of their own, and in ever increasing measure, the means necessary for their work, thus freeing

them from their present dependence on private capitalists, and by facilitating as much as possible the association and spontaneous co-operation of labor it would aim at giving the free-contract regime a much greater development than even at present attained.

The gravest objection that can be made, and one in fact that has been made, is derived from the distrust that all economists feel towards public bodies in general, and the State especially, as regards their capacity for even the administration of those possessions which would be gradually nationalized. There can never, therefore, be too much care exercised in reducing to a minimum these administrative functions especially on the part of the State.

Thus, for example, the administration of the lands gradually nationalized should be entrusted by the State to the Provinces, or to self-governing bodies with a provincial jurisdiction, and these lands should be let either to agricultural co-operative societies or even to individual agriculturists, provided they are united in associations for the purchase of materials, for the manufacture of given products (associated dairies, wine producers, etc.), and for the sale of all the products.

The administration of town properties would be entrusted to the municipal authorities; and it is well known that when old buildings have been pulled down the house property of certain towns has been notably increased by the new buildings, especially in England, and that their administration is carried out in a satisfactory manner.

In any case there is nothing to prevent the administration of these town properties being eventually entrusted, if thought well, to these private companies, which now administer them on behalf of private proprietors.

The mines, the factories and the capital invested in them would come under the control and management of the great national trade-unions of workers of the respec-

tive branches of industry, as by degrees the State handed over to these national trade-unions, under certain conditions of State participation in the profits, the shares inherited by it. This would unite the advantages of co-operative production with those of industry on a large scale: greater interest in the work, as in co-operative production, because the workmen would enjoy the fruits of it in always fuller measure, and at the same time great manufacturing potentiality, which co-operative production has never hitherto possessed, and great discipline, which it has never succeeded in obtaining. And this because those who direct each factory, to whom would be entrusted the maintenance of the discipline, would be appointed—when the number of the respective shares nationalized would come to exceed that of the shares still in private hands—not by those only who worked in the factories, as has always been the case in the manufacturing cooperative societies, but by the whole national trade-union, whereof the workers in this or that factory would be but a small minority. Moreover, the admission of the trade-unions to the control and management of these industries—industries, be it noted, already completely organized and in full working order—would take place gradually with the increase of the number of the trade-union shares in proportion to those still in private property, so that the trade-unions, that is the respective managers, would have full time to become experienced in the control and management, which at first would continue to be entrusted to the managers appointed by the private shareholders; managers, who would probably be appointed also by the trade-union herself when she would have the majority of the shares.

There would remain but a small number of properties the administration of which would have to be carried out by the State, which would entrust it to a national institute formed for the purpose. The institute—to which would

belong also the supreme control of the administration of all the other nationalized properties, intrusted, as we have seen, to the direct management of provincial or communal bodies or of trade-unions—should, in its relation with the State, only pay over to the treasury the quotas belonging to the State of the incomes of all nationalized properties, whether those administered directly by the national institute itself, or those entrusted to provincial or communal bodies or trade-unions above mentioned; and if rendered completely self-governing and independent from the government, there is no reason why it should not be so organized that it would work as well as other similar State institutes and those of other public bodies in general, which today administer collective capital of immense amounts.

To this institute, finally, would belong also the supreme control of the National Bank of Credit to Associated Labor, which—founded by means of the capitals inherited in money—should be created for the purpose of furnishing the farmers of the nationalized lands and the trade-unions who already managed certain factories the capital necessary for working them.[1]

Although the function properly belonging to the State, or the bureaucratic function in general, be thus reduced to a minimum, we do not, however, flatter ourselves that we have yet succeeded in removing entirely from the minds of free trade economists their prepossessions as to the incapacity of the State to perform even purely administrative functions. But this objection, though certainly of a certain gravity, does not seem to be one that cannot be overcome, and for the purpose of overcoming it the thought and ener-

[1] For further details, we must refer the reader to our recent book, published by Zanichelli in Bologna. *Per una riforma socialista del diritto successorio*, which treats the question in all its bearings, and in which, besides quoting the principal criticisms urged against our answers thereto, we have also added the concrete draft of the bill that might be presented to Parliament for the effectuation of the proposal. A French edition will soon appear by the publisher, F. Rieder et C., Paris, with préface by M. Albert Thomas, the leader of the French Socialist party.

gies must be directed of all those who are convinced that no mere difficulties of a technical sort can possibly suffice to arrest the now irresistible and overwhelming onward movement of the working masses, firmly resolved on arriving at the nationalization of all the instruments of production and of all capitals in general.

To sum up: This reform in the law of succession, which would finally permit of the beginning of the much desired nationalization, by pacific and legal but at the same time rapid means, might and ought to represent, we think, that medium programme of socialistic action that is capable of reuniting the Socialist party, now completely bewildered and divided, or at least of attracting to itself the very large majority of the working and popular classes. At the same time, from the legal and gradual manner in which it would guarantee the passage from the old to the new regime, it might not encounter any very bitter resistance from the more advanced and more clear-sighted of the monied classes, who are well aware that to save our civilization from the immense ruin of a violent Bolshevik revolution and to make way for new and more flourishing prosperity, it is necessary to accept the fundamental postulates of Socialism, which, because in accordance with the supreme principles of equity, are not to be combated by any civilized society except at the peril of its own existence.

EUGENIO RIGNANO.

MILAN, ITALY.

THE RELATION OF SPACE AND GEOMETRY TO EXPERIENCE*

I. Geometry as a Schematization of Experience

GEOMETRY is considered by every one to rank among the most certain of sciences. One can have grave doubts, for example, as to the universal validity of any theory in biology, or even honest misgivings concerning the absolute precision of the law of the conservation of energy, but it is hard to imagine a man who is really sincere in questioning the theorem of Pythagoras, that the square on the hypotenuse of a right-angled triangle is equal in area to the sum of the two squares on the legs of the triangle. This conviction which we possess that the theorems of geometry are valid seems essentially independent of any confirmation or substantiation by experience. After we are really initiated into the processes of geometrical reasoning, our certainty of the truth of the theorem of Pythagoras cannot be augmented nor diminshed one jot nor tittle by any actual measurement of a figure illustrating the theorem, if the figure should not substantiate the theorem, so much the worse for the figure, we should say.

It is a highly significant fact, however, that this very science of geometry, which seems to keep itself so independent of experience, is one of the most useful of all sciences in our daily life of experience. The surveyor, the navigator, the carpenter, all make continual use of geom-

* This sequence of lectures was read at Harvard University in the Fall Semester of 1915.

etry in the course of their every-day pursuits, and not only do they do so, but they have an implicit confidence, which always proves to be justified, that the results of their geometrical reasonings—provided only that these are correct in a purely intrinsic, geometrical sense and are based on correctly gathered data—will lead them to perfectly correct conclusions with regard to the world of things experienced with which they deal in their daily lives. The surveyor knows that if his observations are correct, and if he has committed no error of geometry in his computations, the map which he has designed in accordance with a few elementary geometrical laws will be a good map of the region it represents. We have thus the interesting spectacle of a science which seems to scorn experience as its basis, yet furnishes results of the utmost empirical application and value. The question at once occurs to us: How does this happen?

Several theories of the nature of geometry have been devised to bridge this gap. Let us first consider Kant's discussion of geometry. I do not propose to consider here the whole of Kant's treatment of this topic, but only a certain aspect of it—that aspect, namely, which is expressed in the following passage:[1] "Geometry is a science which determines the properties of space synthetically, and yet *a priori*. What, then, must be our representation of space, in order that such a cognition of it may be possible? It must be originally intuition. . . . But this intuition must be found in the mind *a priori*, that is, before any perception of objects, consequently must be pure, not empirical, intuition. For geometrical principles are always apodeictic, that is, united with the consciousness of their necessity, as, 'Space has three dimensions.' But propositions of this kind cannot be empirical judgments, nor conclusions from them."

[1] *Critique of Pure Reason*, Transcendental Aesthetic, §3, Meiklejohn's translation.

That is, Kant says not only that geometry is known *a priori,* but also that our whole original knowledge of *space,* the subject-matter of geometry, is *a priori,* and he regards these two assertions as practically tantamount to one another. It seems to the casual observer, however, as though spatial properties could also be given to us *a posteriori,* in experience. It seems as if the straightness of a stick or its length were known quite as empirically as its color or its hardness. Whatever we may say about space, there is no question possible with regard to the statement that spatial qualities are capable of being experienced. Now, it is not with space in any ulterior sense, but with *spatial qualities* that geometry, as used by the surveyor or the navigator, deals. It is not lines in any purely abstract meaning of the term, but the hair-lines in his telescope, or the path of a light-ray, that concern him, and he knows that if his measurements of the lengths, straightness, angles, etc., of these are correct, his computations will also be correct, provided only that he has made proper use of geometrical reasoning. It is such lines as these that form part of his space—and yet he feels the need of no experiment to substantiate the result of his geometrical reasoning. The *a priori* certainty which Kant attributes to geometry is one which is utterly irrelevant to its applications in our life; the abyss between his space, to which geometry applies, and the concrete spatial properties of concrete things, remains unbridged in his system, notwithstanding the fact that he calls space the form of our external experience since the apriority of the geometry which we apply must be the apriority of an empirical intuition, not that of a pure intuition. The geometry which he discusses is one which applies to an entirely non-empirical realm, and which he nowhere brings into touch with those fields of experience in which our every-day geometry plays so great a role.

One of the chief motives which leads Kant to this somewhat incomplete if not positively unsatisfactory treatment of geometry, as one can readily see from this paragraph which we have quoted, is that he considers the apriority of geometry impossible unless our knowledge of its subject-matter is also *a priori*. It is clear, then, that *if we can consistently hold that it is possible for geometry to be a priori, and yet to have an empirical subject-matter*, one strong argument in favor of Kant's view of space has vanished, and we are able to formulate a theory of the relation of the non-empirical science of geometry to the objects of our experience as surveyors or navigators, etc., which is more consonant with the views of our every-day common sense than that of Kant. It is this view of the relation between experience and geometry— the view, namely, that geometry, though *a priori*, deals with an empirical subject-matter— which I intend to suggest as a possibility in what follows.

Before I go on, however, to my discussion of this theory, I wish to devote a little time and attention to a third theory, different both from that of Kant and from that which forms the thesis of this course of lectures. This theory is that of Ernst Mach, as expounded in his little book, *Space and Geometry*.[2] Professor Mach's views form the precise antithesis of those of Kant, both with respect to space and to geometry. As to space, he says:[3]

"If for Kant space is not a 'concept,' but a 'pure (mere?) intuition a priori,' modern inquiries on the other hand are inclined to regard space as a concept, and in addition as a concept which has been derived from experience. We cannot intuit our system of space-sensations *per se;* but we may neglect sensations of objects as something subsidiary; and if we overlook what we have done, the notion may easily arise that we are actually concerned

[2] Translated by T. J. McCormack, Open Court Publishing Company, Chicago.

[3] *Op. cit.*, p. 34.

with a pure intuition. If our sensations of space are independent of the quality of the stimuli which go to produce them, then we may make predications concerning the former independently of external or physical experience. It is the imperishable merit of Kant to have called attention to this point. But this basis is unquestionably inadequate to the complete development of a geometry, inasmuch as concepts, and in addition thereto concepts derived from experience, are also requisite to this purpose."

Mach claims, in other words, that space is essentially a system of *space-sensations* or *space--experiences*, which seems to take the form of a "pure intuition" merely because in our geometrical considerations we confine our attention to one particular phase of the objects with which we are concerned, and neglect all those aspects of our experiences which, though they are necessarily present, are not spatial in their nature. According to him, he says, we are enabled thereby to consider the interrelations of the spatial aspects of our experience with entire disregard of what the other sides of our experience may be. Nevertheless, he holds, space is given to us in a completely empirical manner. Or, as Mach puts it in another book of his,[4] "Space and time are well-ordered systems of sets of sensations."

It seems obvious to the common-sense of us all that Mach is at bottom correct in this statement, for space is somehow or other, we all should say, a system of experiences. Everything looks promising, therefore, for a satisfactory account of the sources and nature of our geometrical certainty. Let us see what the explanation of this is which Mach offers us. He expounds his view as follows:[5]

"The knowledge that the angle-sum of the plane triangle is equal to a *determinate quantity* has thus been reached

[4] *The Science of Mechanics*, translated by T. J. McCormack, Open Court Publishing Co., p. 506.
[5] *Space and Geometry*, p. 58.

by experience, not otherwise than the law of the lever or Boyle and Mariotte's law of gases. It is true that neither the unaided eye nor measurements with the most delicate instruments can demonstrate *absolutely* that the sum of the angles of a plane triangle is *exactly* equal to two right angles. But the case is precisely the same with the law of the lever and with Boyle's law. All these theorems are therefore idealized and schematized experiences: for real measurements will always show slight deviations from them. But whereas the law of gases has been proved by further experimentation to be approximate only and to stand in need of modification when the facts are to be represented with great exactness, the law of the lever and the theorem regarding the angle-sum of a triangle have remained in as exact accord with the facts as the inevitable errors of experimenting would lead us to expect; and the same statement may be made of all the consequences that have been based on these two laws as preliminary assumptions."

This result—namely, that Mach regards the certainty of geometry as of empirical origin, and simply due to the fact that our experiments with lines and angles, etc., by means of paper-folding and similar methods have always substantiated our geometrical predictions as well as could be expected when we take into consideration the inherent inaccuracies of the experiments—this result, I say, is by no means satisfactory. Nobody would ever think of testing the theorems of Pythagoras by means of a foot rule or a protractor; the only things which would be tested by such an attempt and which would have to be rejected in case of a non-verification of the theorem would be the foot rule or the protractor. However useful paper-folding and similar pursuits may be in leading our interest toward things geometrical and in giving us the first dawning ideas about what it is with which geometry concerns itself, geom-

etry deals *directly* with points, lines, planes and angles, and not, except in some periphrastic sense, with such gross topics as folded bits of paper, rules, and micrometers. Whatever the edge of a piece of paper may do or be, a *line* is the shortest distance between two points, does not cut any other line in more than one point, and has all the other properties which are attributed to lines in a text-book of geometry. If a crease in a piece of paper fails to have these properties, why—it simply is not a line. However useful geometry may be in the theory of paper-folding or navigation or astronomy, *prima facie* geometry is *not* the study of paper-folding nor of navigation nor of astronomy, and the accuracy or correctness of any part of any of these studies may be impeached without involving as a corollary the impeachment of any portion of geometry or theorem belonging to it. The geometry of which Mach talks is simply not the geometry of the mathematician; Mach solves the problem of space and geometry to his own satisfaction by flatly ignoring the non-experimental nature of geometry, just as Kant solves it by not entering into a discussion of that empirical character which actually pertains to space. Both positions are unnatural; what is the natural alternative which avoids the objections besetting each of them?

I have already stated that the view which I maintain in this course of lectures is that geometry, though *a priori*, deals with an empirical subject-matter. How is this, however, possible? How can our study of a subject which is known in a manner open to all the uncertainties and inaccuracies which beset empirical knowledge in all its manifestations—namely, space—be possessed of an *a priori* and purely intrinsic certainty, not rooted at all in experience? The answer to this question is by no means as difficult as it might seem at first sight. It will be noted that Mach does not make geometry deal with raw, undi-

gested experience, but, as he says, "All these theorems are . . . idealized and schematized experiences."

Now, the study of an idealized or schematized experience differs from that of a raw or crude experience in that it has to take account of two distinct factors—the experience, and the mode of schematization employed. To illustrate how this is the case, suppose that I am considering a set of statistical tables of the death-rate of Boston from year to year. I may regard these tables from several different standpoints. I may be interested, for example, in the seasonal fluctuations of the death-rate. In this case the table of statistics gives me information which could not have been predicted with more than approximate accuracy and certainty, and which is completely dependent upon concrete experience. On the other hand, I may be primarily interested in the method of tabulating statistical data which is used in these tables; in this case, when I have once grasped the principle underlying the method, I am quite as well able to predict anything you please in the next year's tables which concerns details that are dependent solely on the method of tabulation employed as I am to yield the same information concerning this year's tables or concerning last year's tables. The method of tabulation employed may and should be made as suggestive as possible of the actual empirical laws of the death-rate of Boston and as useful as possible in the handling of the data tabulated, but once it has been chosen, it is entirely independent of the particular empirical properties of these data, and remains essentially incapable of substantiation or of contradiction by them. Thus, though the study of his tables from the standpoint of the form of tabulation employed is of immense practical use to the statistician for the handling of his empirical material, once that form is definitively fixed, it is really an *a priori* science, notwith-

standing the fact that the data expressed in the tables are themselves known *a posteriori.*

It is possible to regard geometry in a way quite parallel to a set of statistical tables—though I do not mean to suggest that statistics play any part whatsoever in geometrical reasoning. We may regard a point, for instance, not as a direct object of experience, but as a certain arrangement or collection of objects of experience, in a manner which I shall explain in detail in the subsequent lectures of the present course. A point of this sort will, in general, depend for its actual properties on the concrete natures of the experiences of which it is constructed, but it will also have certain properties which, unlike its other attributes, are independent of the concrete natures of these particular experiences, and are predictable on the basis of a knowledge merely of the principle in accordance with which the points of our space have been synthetized from our experience. These latter properties of points are studied in geometry, while those which are dependent on concrete experiences belong rather to physics or to the other natural sciences. Thus space, which is made up of points, lines, etc., constitutes a kind of tabulation of the experiences of our outer senses; yet geometry, which has space as its subject-matter, since it depends on the method of tabulation alone, as I claim in this course of lectures, is an *a priori,* not an experimental, science. This is the view for the possibility of which I am here pleading.

My view might be stated as follows: Geometry is the science of a *form* into which we cast our spatial experiences. I shall not express my view in this manner, for I wish to keep it clearly distinct from two other views which might with equal justice lay claim to this mode of expression. These views are that of Kant, upon which we have already touched, on the one hand, and the view of those mathematicians, on the other, who hold that the only spe-

cies of geometry which can possess *a priori* certainty is that geometry which concerns itself, not with the actual points and lines of the world in which we live, but with the *laws* in accordance with which a great many of the properties of these points and lines can be deduced from a small number of properties which they seem to possess, or at any rate seem to possess approximately.

Let us first see wherein our view differs from that of Kant. Kant says that geometry is the synthetical science *a priori* of the form of the external sense, whereas we say that geometry deals with the intrinsic properties of a schematism into which we cast our external experiences; wherein lies the real difference between these two very similar views, and what is its significance? The difference is this: Kant regards geometry as the study of a schematism imposed on the world by our external senses themselves, before any act of experience, and utterly independently of any such act. On the other hand, we maintain that geometry deals with an experience schematized after it has come into existence, and with concrete practical ends in view, even though this schematism may be permanent once it has come into existence and been accepted by us. As a consequence, Kant is unable, as we have previously indicated, to explain how it is that we are able to apply geometry to experience *in a certain concrete and definite manner*, as it is applied by the sailor and the surveyor, or at least he fails to give any hint of how this application is to take place, for the schematism which constitutes the subject-matter of geometry is made, he tells us, before and without reference to the concrete experiences of the surveyor and the sailor, by the essential nature of the outer senses, themselves, and would be the same were there no such particular experiences as those of the sailor or the surveyor. We, on the other hand, are able to maintain that the schematism of geometry is useful for the sur-

veyor and the sailor just because it is designed with the purposes of the surveyor and the sailor in view. This is still true even though that schematism remains just what it is forever, once it has been selected. For example, we choose the schematism "line" in such a manner that some particular line of geometry will be determined as unambiguously as possible in a certain easily recognizable manner by every ruler edge or plumb-line or line of vision in our actual experience. Then, while the lines we have chosen in our schematism may have a host of interesting and valuable properties which are determined by the schematism alone, we may make certain of our geometrical objects standing hostage, as it were, for the physical objects mentioned above, and make our reasonings and experiments refer to these lines rather than to the physical objects themselves, so that our reasonings and experiments may be facilitated by the manifold transformations and systematizations suggested by pure geometry. Since our geometrical lines, though constructions and schemata, are constructions and schemata made on the basis of concrete experiences, we are able to recognize empirically this correspondence between geometrical lines and certain physical entities to which we have just referred, and hence make the former take the place of the latter in the formulation of scientific laws. This cannot be done on the basis of Kant's theory—and this is its fatal defect—because space, according to him, though the form of our external experience, is completely prior to any concrete experience, and hence no correspondence between certain spatial entities and certain physical entities can be recognized empirically, if we accept his theory of the matter.

So much for Kant; let us now consider the pros and cons of the view of those who hold that the only sort of geometry which can possess *a priori* certainty is that geometry which concerns itself, not with the actual points and

lines of the world in which we live, but with the *laws* in accordance with which a great many of the properties of these points and lines can be deduced from a few laws which we observe that these points and lines possess, or very nearly possess. This view, that is, says that the real subject-matter of geometry is the formal deduction of its theorems from its axioms, which are not self-evident statements concerning the space in which we live, but mere hypotheses which may perchance be satisfied by an infinity of systems, and it claims further that geometry is not at all concerned with the question whether these axioms and theorems apply to any particular objects or constructions in the world of sense. This latter application, it maintains, must be determined by experience alone, and depends on experience for its validity. Now, it is perfectly true that there is a legitimate non-empirical science, which has as good a claim to the name of geometry as the discipline which we are discussing here, which is concerned with the deduction of the theorems of Euclid from the axioms of Euclidean geometry. I doubt, however, whether this mere abstract logical deduction constitutes the whole of what we ordinarily call geometry, or even the whole of that part of geometry which can lay claims to *a priori* certainty. There certainly appear to be such things as lines, which are more than mere blank spaces in the scheme of symbolism or of logical deduction by means of which the appropriate theorems are obtained from any set of truths which can be put into the form expressed in Euclid's axioms. It seems as if these lines must, from the very necessity of their nature, satisfy the laws of Euclidean geometry, while certain particular lines bear an intimate association with such concrete empirically known things as straight edges and light-rays. This association seems to be presupposed in our every-day life when we say, "This is more nearly a true line than that," as if the true line

were a sort of a criterion with which we could empirically compare certain empirical objects. This two-faced aspect of geometry, which is *a priori*, yet deals with an empirical subject-matter, is not explained by those who hold the view we criticize, and is explained on our view.

We hold, then, that geometry is an *a priori* science, which deals with a certain schematization of experience, which we may call space, in so far as its properties depend on the method of schematization alone. This schematization has a superficial appearance quite different from that of the experiences of which it is composed before they are schematized. Experience presents us only with objects that have extension, while a point has no extension. Experience never gives us a perfectly straight line, nor a precise circle, nor an absolutely accurate sphere. All these things, however, form topics dealt with in geometry. Now, we have claimed in this paper that geometry is a schematization of experience, not in the sense that it is a kind of approximate copy of experience with all the roughnesses left out, but in the sense that it is formed from experience by the application of some principle, just as a table of statistics represents the facts it concerns in accordance with a certain principle of tabulation. Just as, notwithstanding the fact that a table of statistics does not resemble the matters tabulated, a statement about the former is but a periphrasis for a statement about the latter, so a geometrical proposition is really concerned with experience, notwithstanding the fact that its direct subject-matter has an appearance differing in many respects from that of experience.

After all this talk of geometry as a method of tabulation, many of you will want to see a concrete example of this sort of tabulation, taken from the field of geometry. It is rather difficult, however, to exhibit such an example in the limited portion of this lecture which remains. I

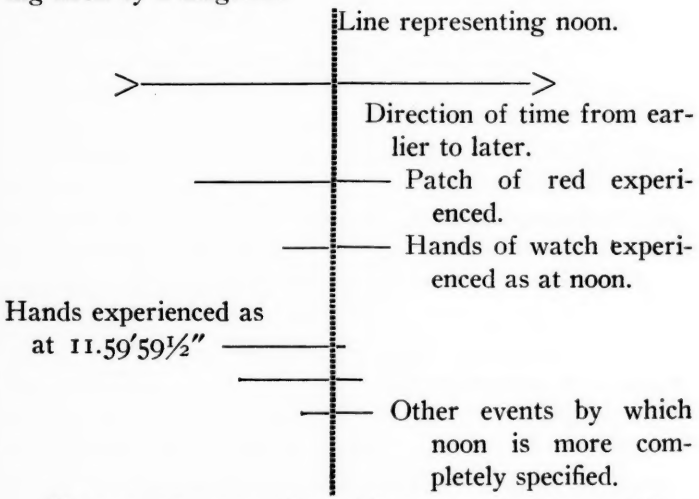
can, nevertheless, give you an example of a similar tabulation employed in a field very analogous to geometry—the study of the formal properties of time.

There are certain laws which we always unquestioningly accept as valid concerning time in quite the same spirit that we hold geometry to be *a priori*. We believe, for example, that time is composed of instants which are timeless, that no two instants are contemporaneous, that of two distinct instants, one must precede the other, and that if the instant *a* precedes the instant *b*, and the instant *b* precedes the instant *c*, that *a* precedes *c*. We consider these statements as quite as truly *a priori* as the theorem of Pythagoras, and regard the former and the latter as quite analogous with one another. The events which we experience, however, always occupy time, an event may neither precede nor follow another, and so on indefinitely. How are we able, the question is, to regard instants as tabulations of events of such a sort that we can be sure, from a knowledge of the method of tabulation alone, without any concrete empirical knowledge of events, that instants will have the formal properties we have attributed to them? I shall give such a method of tabulation in the following paragraphs, though I cannot spare the time to show, as is the case, that it is a consequence of the method itself that instants have the formal properties we have attributed to them.

Experienced events are said to happen at certain instants: what do we mean by such statements? When I say that I see this patch of red here at, say, noon, what do I mean? My first meaning is, perhaps, that I have taken out my watch, looked at it, and have seen both the hands

pointing at the figure XII, and that I have experienced this as simultaneous with my experience of the patch of red. But if I look into the matter more thoroughly, I find that this is not all I mean to assert when I say that I see this patch of red at noon. The time during which I see the hands of my watch in a certain position is always of a duration not zero. If the watch had suddenly passed out of existence while I was looking at it, I should still have continued to have seen it for a fraction of a second, during which noon would have passed beyond recall. Now, I can approach more nearly to a precise formulation of what I meant by the proposition that I saw the patch of red at noon if I name still other events which were experienced as simultaneous with the position of the hands of the watch, but which did not endure in experience for the whole period that the hands of the watch were experienced to remain in their position. For example, I can say, perhaps, that this patch was not only experienced as simultaneous with the position of the hands indicating noon, but that the experience of their indicating 11.59′59½″ had not yet died out while I saw the patch. By noting more and more events, each experienced as simultaneous with the patch, and each experienced as simultaneous with each other (for they all, we should say, are experienced as being at noon), we can finally arrive at the specification of a given instant without duration at which the patch was seen, though all the events used in fixing this instant may have consumed time, and have been possessed of all the other gross properties characteristic of *experienced* events, and the relation of simultaneity among them may have been given in experience.

Perhaps I can best illustrate our method of determining noon by a diagram.


Line representing noon.
Direction of time from earlier to later.
Patch of red experienced.
Hands of watch experienced as at noon.
Hands experienced as at 11.59′59½″
Other events by which noon is more completely specified.

We are able to continue this process further and further by the adjunction of more and more events to the set by which we determine noon. Every such event will be experienced as simultaneous with every other. Finally we shall come to a stage where no more new terms can be found which we can adjoin to our set—that is, there will be no other events which will be experienced as simultaneous with *all* the members of our set. In such a case, we shall have given as complete a determination of noon as is possible on the basis of experiences. The patch of red, which we wish to say is seen at noon, is one of these.

But what *is* noon, which seems to form the subject-matter of the proposition, "This patch of red is seen at noon," which we have been considering? We wish to interpret noon as a sort of tabulation of experience: the answer we give shall therefore read, "Noon is the whole class of events, each of which is experienced as simultaneous with each other, which contains every event experienced as

simultaneous with all its constituent events, by means of which we have dated our patch of red." This definition may seem to be circular, for it may seem that the class of events in question could not be specified except with reference to a pre-existing notion of noon. This criticism is, however, invalid, for it may be shown that if instants and events have the formal properties and interrelations that we universally attribute to them, every instant, such as noon, will determine uniquely and be uniquely determined by some set of events of which every two members are experienced as simultaneous, and which is such that it contains every event which is experienced as simultaneous with all its members. The definition of these latter entities involves no circularity, for it depends merely upon a previous acquaintance with events, the relation of simultaneity and a few elementary logical notions such as that of a collection, and not at all upon any specific acquaintance with noon, or with any other instant.

Now, the relation of simultaneity among events, whether as such it can be experienced or not, is certainly far closer to experience than an instant. In all this work, the point we are making is not that the terms and relations with which we start and which we take to represent experience are immediately given—we do not even assume that there are immediately given terms and relations—but that, if I may use such a phrase, they are closer to givenness, that they are less elaborated, that they are the results of a lesser degree of sophistication than the ordinary notions of science. Whether the "experience" with which we started in this lecture is itself already schematized or not does not concern us here; it is enough that space and time mark a degree of schematism greater in intricacy than what we here call experience.

To sum up: we have contrasted the aloofness of geometry from empirical verification with its tremendous value

when applied to experience, and have noted the problem which this situation creates. We have discussed Kant's views on the relation between geometry and experience, and have seen that his statement that geometry deals with space, which is given prior to any concrete experience whatever, is hard to reconcile with our empirical recognition of geometrical forms. We have seen that a view which should hold that geometry, though *a priori*, deals with an empirically known subject-matter, would avoid this particular difficulty. Then, taking up Mach's standpoint, it became clear to us that his view, that geometrical certainty is of experimental and empirical origin, is in direct conflict with the practice of all mankind in matters geometrical, and that we all should hold that any geometrical experiment was a test rather of the instruments of measurement used than of the geometrical theorem involved. We noticed the suggestiveness of Mach's notion of geometry as the science of a schematized experience, but saw that in the study of statistical tables, for instance, certain aspects of the study of a schematized experience may be independent of the matter schematized, and depend only on the form of the schematism. We held geometry to be of a similar nature. We observed that our view lent itself to the formulation, "Geometry is the science of a *form* into which we cast our spatial experiences," but we observed that such a formulation would also cover Kant's view that geometry is the study of the form of the external sense, and the view that geometry is merely concerned with the deduction of geometrical theorems from geometrical axioms, so that the certainty which we usually attribute to geometry is entirely dependent on the fact that it is a science of abstract deduction. Kant's view, we found, differs from our view in the fact that it makes the form of the external sense prior to all experience, and consequently cannot explain the empirical identification of spatial entities, while we hold that the

schematism with which geometry deals is imposed only after and on the basis of concrete experience. The view that geometry is only concerned with a certain deductive chain did not explain why we act as if geometry were the *a priori* study of a certain concrete system which we can apply to experience as a criterion of straightness or of circularity or of any similar geometrical property. We saw that geometrical propositions, though they seem to deal with such entities as points, lines, etc., are mere paraphrases for propositions about experience in some more direct sense. We finally gave an example of the sort of schematization or tabulation of which geometry makes use, taking this example from *Our Knowledge of the External World as a Field for Scientific Method in Philosophy,* by Mr. Bertrand Russell.

The remaining lectures will be devoted to a more or less tentative discussion of the details of the methods of tabulation and schematization used in geometry. They will very often involve the use of simple geometrical reasoning, but only to prove that the methods of tabulation we here employ yield results similar to those yielded by the methods of schematization which we must tactily use in building up the entities of our every-day geometry.

II. The Point as a Tabulation of Solids

In our last lecture we put forward the view that geometry is concerned with the study of a certain tabulation or arrangement of experience, in so far as this arrangement is determined, not by the nature of the material arranged, but by some already fixed principle of arrangement. As an example of what such a tabulation or arrangement would be like, we gave a brief discussion of the definition of instants as arrangements of simultaneous events. In this and the following lectures we shall attempt, in a similar manner, to define the subject-matter of every-day geometry as a system of tabulations of things which can be experienced and their relations—that is, to exhibit the methods of schematism employed in geometry. In our last lecture we made the further claim that the ordinary theorems of Euclidean geometry could be regarded as consequences solely of the methods of tabulation and arrangement employed in geometry. In the ensuing portion of this course we shall try to show, as far as we are able, how points and the relations between points may be so defined as complexes of objects which can be experienced and of their empirically knowable relations that, though space will be dependent on sense, the geometrical properties of space will be independent of sense, and will follow solely from the schematism by which space is obtained from sense. We shall aim to show that, just as a cube does not depend for its cubical properties on the material from which it is made, just as a wooden, a stone and an iron cube all have eight apices, twelve edges and six faces, so a geometry, although its propositions may have relevance to the actual world in which we live, has a validity independent of the particular

nature of the world to which it applies. We maintain that geometry has this universal validity, not only in the sense in which it says that *if* any system satisfies a certain set of premises geometry is applicable to it, but also in the sense in which it asserts categorically that geometrical theorems must apply to the entities which we define as points, whatever the concrete nature of the world in terms of which they are defined may be.

The first task which we have before us is the determination of the fundamental spatial experiences in terms of which our subsequent schematizations and definitions are to be made. The first essential condition which these fundamental experiences must satisfy is that they should be genuine experiences. This excludes at once the possibility that they should deal with such essentially non-experienceable entities as points without magnitude or curves without thickness and so forth. It demands that the fundamental spatial experiences should concern such things as visible patches of color or tangible solids. This necessary condition which these experiences must satisfy leaves us still a great possibility of ambiguity as to their nature. As we are humanly unable to do what is perhaps the most natural thing in this situation and make our method of schematization apply to all experiences which we should ordinarily claim to have a spatial import, on account of the immense technical difficulties such a task would involve, we are obliged to introduce a certain degree of arbitrariness and artificiality into the selection of the fundamental experiences from which we shall build up our geometry. Whether the experience of the solid be primitive in experience or not, this much is certain, that it belongs to a much lower stratum of schematization and synthesis of experience than such unextended things as points, lines and other geometrical entities, and that things or solids are the last word in primitiveness and immediate givenness

for the man unsophisticated by psychology. Since our discussion in these lectures is only tentative anyway, and since solids offer a very convenient starting-point for the development of a schematism leading to geometry, we shall regard our primitive spatial experience as one dealing with solids. We have not yet completely specified the nature of our primitive spatial experience, however, as it is possible that there are many different kinds of facts concerning solids which can be experienced. One of the simplest to handle—although possibly not one of the simplest in the order of experience—of these facts is the fact that a certain solid is observed to contain a part in common with another solid. We shall, therefore, select an experience of the intersection or overlapping of two solids as the fundamental spatial experience. Two solids, we shall say, are experienced as intersecting or overlapping or having a part in common with one another if they both seem to contain some solid or if one seems to contain the other or if they seem to come into contact.

The experience of the overlapping of solids is not, however, as it stands, a sufficient point of departure for a schematization which is to lead to geometry. We wish to be able to define a straight line as a sort of a tabulation of solids. Now, if all we know about solids is the relations of overlapping that hold among them, we will be unable to discriminate between a straight line and a tortuous one. The whole of space could be kneaded like a lump of putty without changing a solid into anything else or altering the relations of overlapping which solids bear to one another, but by such a transformation you could deform a straight line into a curve as tortuous as you please. It is obvious, then, that if we are to be able to define straight lines in terms of the experience of the intersection of solids, we must put some kind of a limitation on the kind of solids considered. We shall put upon them the limitation that

they are to be *convex*. Now, a convex solid is one such that any two points which belong to it can be connected by a piece of a line which nowhere passes outside of it. Thus a solid sphere is convex, a cylinder is convex, a cone is convex, and a cube is convex, while a solid in the shape of an hour-glass is not convex, a doughnut is not convex, a bowl is not convex, and no figure which is hollow is convex. As a matter of fact, convexity is synonymous with the absence of hollowness in any sense, and since hollowness can roughly be judged by the eye and the finger without reference to straight lines, convexity may also be determined by a more or less direct reference to experience. We know what it means to say that a bowl is hollow and that a billiard-ball is convex long before we ever think of correlating these properties of solids with the definition of convexity just given. We can, therefore, make our fundamental experience that of the intersection of *convex* solids, and be sure that it is near to genuine experience. Further, it may be shown by a simple bit of geometry that if the world were, say, made of clay, and were so squeezed out of shape that all convex solids and their relations of overlapping should remain unchanged, every straight line would remain straight. Consequently, once the set of all convex solids in space has been identified, the set of all lines in space is determined, and it seems very probable, to say the least, that lines can be defined in terms of convex solids.

The sort of fact from the schematization of which we shall obtain space is, "This convex solid is *experienced* to intersect that one," and not simply, "This convex solid intersects that one." The formal properties of experienced intersection and of actual intersection are probably, however, closely analogous in most respects. Each solid may be regarded as having, outside of its physical extension, a sort of *aura*, of definite extent, such that two solids are experienced as overlapping when, and only when, the solids

formed out of each by adjoining to it its aura actually intersect. For example, two spheres a hundredth of an inch apart may seem to be in contact, as far as our unaided senses can tell: then we shall say that the aura of each extends at least one two-hundredth of an inch beyond its physical extension. The difference between the relation of apparent or experienced intersection among convex solids and that of their actual physical intersection is to all intents and purposes, then, a difference in the solids chosen as intersecting rather than in the formal properties of the relation of intersection itself, for if we replace convex solids by convex solids plus their auræ, we can interpret the apparent intersection of the former as the actual intersection of the latter.

We are now in a position to define our points—that is, to exhibit them as tabulations of convex solids. We shall define our points as collections or aggregates of solids. This may seem curious to many of you. "What!" you may think, "Is not a point small and a solid large? Is not a class of solids even larger than a solid? Then how can a point be a class of solids? How can the part be greater than the whole? How can points be made of solids, as you say, and solids also be made of points, as the mathematician says?" Now, all these questions result from a confusion of the relation of a member of a collection to the collection of which it is a member with the relation of an object filling a given space to an object filling a space including that which the first object fills. One tends to think, for example, that because Harvard University is a class of men, Harvard University fills more space than a single man. But, when one comes to think of this example more thoroughly, one sees that *in the sense in which a man fills a certain space,* it is nonsense to talk about Harvard University as filling any space. It is only in a metonymous sense that Harvard University can be said to fill the space occu-

pied by all its members. Harvard University has only such properties as belong to different logical dimensions from those of its members. In fact, it is a general proposition of logic that no collection can have any properties that can in precisely the same sense be significantly asserted—or denied, for that matter—of any of its members. Thus, Harvard University, although it has certain intimate connections with certain portions of space, cannot be said to occupy any space at all in the sense in which I now occupy, the space vertically above this platform, and in an analogous way, in the sense in which a solid can occupy space, a class of solids cannot occupy space, and in the sense in which a class of solids can occupy space, a solid cannot occupy space. It is, therefore, nonsense to speak of a class of solids as either smaller or larger than a solid. Hence we do not, in defining a point as a class of solids, make the part larger than the whole, for the point and the solid are rendered by such a definition incomparable as to magnitude.

The second paradoxical feature of our definition of a point—that we define a point as a collection of solids, whereas in ordinary geometry, a solid is regarded as a class of points—is eliminated still more easily. A solid, in the sense in which points are classes of solids, is an entirely different thing from a solid, in the sense in which a solid is a class of points. They are no more identical than the collection of clubs to which John Smith belongs is identical with John Smith himself. The only thing that entitles us to call both solids is that the world in which we live is probably so organized that corresponding to each solid in our first sense there is a class of points uniquely determined by it and representing no other solid than it, which we may call "the same solid as it," just as it might be that in some town one could identify every man by the list of clubs to which he belongs, and could say, whenever one should

see a list of clubs to which some man belongs, "That's John Smith," or "That's William Jones," or whoever else it might be.

Our definition of points in terms of solids is to be justified, as are all definitions in this kind of work, by its fruits. We shall so define a point that if the things we call convex solids are really the convex solids of an ordinary Euclidean space, the things we call points will correspond in a certain determinate manner to the points of ordinary Euclidean space; the things we shall later call lines will have all the nice properties that lines should have; and finally, the whole space we shall obtain as the end of our discussion will have all the attributes that pertain to our every-day space. On the basis of this first definition of points and of lines we shall give a second and finally a third definition of points and of lines which will, on the one hand, make each point of the first sort determine a single point of the second or third sort and each line of the first sort determine a line of the second or third sort in such a manner that the geometrical properties of a figure made up of points and lines of the first sort will be substantially unchanged if each point and line of the figure be replaced by the analogous point or line of the second or third sort—which will, I repeat, do all this *if the points and lines of our original system form a set satisfying the axioms of ordinary geometry*, or, to put it in a more elementary manner, if two lines in our first sense have a point in common when and only when two decent and well-behaved lines ought to have a point in common. On the other hand, we shall so frame our definitions of points and of lines of our third kind that, however irregular the formal properties of the points and lines of our first sort may be, however often lines that should intersect, did our original system obey the laws of geometry, fail to intersect, or lines that should fail to intersect do

intersect, our lines and points of the third sort must, so long as logic is logic, have all the properties appertaining to lines and to points in a Euclidean geometry. Furthermore, we shall develop a theory of measurement in this third space that we finally attain which will be consonant, on the one hand, with our usual ideas of the operations performed in actual physical measurement, and which, on the other, will be in perfect harmony with the laws of measurement laid down in ordinary Euclidean geometry. All this is done on the basis of our original definition of a point, and constitutes an ample justification for it.

After this rather long-winded apology for the definition of a point as a class of solids, let us state this definition in precise terms. *A point is a collection of convex solids such that* (1) *any two convex solids belonging to it are experienced as intersecting, and* (2) *if a convex solid is experienced as intersecting EVERY member of such a collection, it can only be itself a member of the collection.* We saw previously that the relation of experienced intersection among convex solids reduces itself to the relation of actual intersection among other solids—namely, those formed out of convex solids by adjoining their auræ to them, or as we shall hereafter call them, *a-solids.* Our definition is therefore practically equivalent to one which should read as follows: a point is a class of a-solids such that (1) any two members of the set intersect, and (2) any a-solid that intersects every member of the class must itself be a member of the class. Now, what does this mean?

Let us consider the class of all the a-solids which, as we should say in our every-day life, contain a given point x on their surface or in their interior. In the first place, every two members of this set intersect, for earlier in this lecture we have taken the term intersection to cover contact or tangency, and two figures with a point in common

either intersect bodily if the point in question lies in the interior of one of them, or come into contact with one another if the point lies on the surface of each of them. In the second place, it may readily be shown that if an a-solid intersects every a-solid that contains x, it must itself contain x. This proof depends upon the fact that if an a-solid does not contain a given point, another a-solid can be found which contains the point, but does not intersect the first a-solid. Taking this principle for granted—its truth can very easily be established on the hypothesis that the aural layer of a convex solid is of a uniform thickness throughout space, or on many similar hypotheses which do not assume so much—the desired consequence follows in this way: if an a-solid intersects every a-solid that contains x, but does not itself contain x, we get a contradiction, for by the principle which we have just enunciated, there must be a second a-solid, not intersecting our first a-solid, but containing x, while, by hypothesis, this is impossible. Consequently, if an a-solid intersects every a-solid that contains x, it must itself contain x. We have thus shown that a collection of all the a-solids which, as we should ordinarily put it, contain some point, satisfies both the criteria which a class of a-solids must fulfil to be a point by our definition, since any two members of it intersect, and any a-solid which intersects all its members belongs to it.

To give a completely satisfactory justification of my definition of a point as a class of a-solids whereof any two intersect and which are such that any a-solid intersecting every member of the set belongs to the set, it is not enough to show, as I have just shown, that every collection of all the a-solids containing some point, which may be said to represent or even to be that point, is a point in accordance with our definition; we must also show that no other collections of a-solids are points in accordance with our defini-

tion. We must show that if, on the one hand, a collection of a-solids does not exhaust those which, as we should ordinarily state it, contain some point in common, or if, on the other, there is no point common to all its members, the collection of a-solids in question fails to satisfy one or both of the two criteria which determine whether a given collection of a-solids is or is not a point in accordance with our definition of a point. Now, it is easy enough to show that if all the members of a collection *a* of a-solids contain a given point, but do not exhaust the collection of the a-solids which contain the point, there are other a-solids—i. e., the other a-solids containing the point in question—which do not belong to the collection *a*, but intersect every member of *a*, so that *a* is not a point in accordance with our definition. It is not easy to show, however, that if a collection of a-solids is of such a nature that there is no point, to use ordinary geometrical language, which all its members contain in common, this collection of solids fails to satisfy at least one of the two criteria both of which a collection of a-solids must satisfy if we are to call it a point in accordance with the definition we have given. In fact, I have not yet succeeded in proving this theorem, and I have nowhere seen any proof given for it, yet I am convinced that it is true and that it can be proved. I am convinced of this because, notwithstanding a considerable amount of effort, I have been unable to discover a single collection of a-solids, except the collection of all the a-solids that contain some given point, which satisfies both of the two conditions which all the things that are points by our definition must satisfy. Therefore, notwithstanding the gap in my chain of reasoning, I shall go on from this point as if I had proved that our definition of a point is perfectly adequate, and that the only collections of a-solids which satisfy our definition of a point are such as are made up from all the a-solids which,

as we should naturally put it, contain some point. If we suppose that this is proved, *provided that our experience of the relation of intersection among convex solids is to receive the geometrical interpretation in terms of a-solids which we have given it,* since our first definition of points in terms of the experience of the intersection of convex solids will then be practically equivalent to our second definition of a point in terms of a-solids, our points in our first sense, though defined in terms of an experience, will well deserve the name of points.

Our next task is to define what is perhaps the next most fundamental notion in geometry—the notion of a line—in terms of our experience of the intersection of convex solids. It will be remembered that convex solids stand in a very close relation to straight lines, for a convex solid is one that contains the whole of a bit of any straight line whose ends lie inside it. Now, this fact enables us to define a bit of a straight line in terms of our experience of the intersection of convex solids as follows. We have just seen how a point may be regarded as a class of a-solids which is what we should ordinarily call the class of all the a-solids containing that point. An assumption which we shall make at this point is that all a-solids are convex and that we can thus regard a point as a class of all of a certain kind of convex solids which contain a given point. This assumption is extremely natural. It is a consequence of the other assumption which we suggested previously, to the effect that the aural layer of a convex solid is of uniform thickness throughout space, but does not presuppose the latter assumption. From the hypothesis we have stated we can readily draw the conclusion that if a and b are any two points *qua* classes of a-solids, then every a-solid which forms a member both of a and of b contains, in ordinary geometrical phraseology, the whole piece of a straight line intercepted between a and b. That this is true follows from

the fact that *a* and *b* are points inside any a-solid which belongs to them both, since a member of a point is an a-solid which contains it. Consequently, since an a-solid is convex, any a-solid which belongs both to *a* and to *b* contains the whole linear segment or bit of line between them, and consequently every point on this segment. Therefore, every point on this segment possesses as a member any a-solid within which *a* and *b* lie. This is another application of the principle that the members of a point are the a-solids which spatially contain it. It is thus a necessary condition if *c* is to lie on the linear segment between *a* and *b* that all those a-solids which belong both to *a* and to *b* should also belong to *c*. That this condition is also sufficient may be proved on the hypotheses that the thickness of the aural layer of all a-solids is constant and that an a-solid can be transported to any part of space, and yet remain an a-solid. Both these hypotheses are very probably true—at least within that part of space whereof we have any experience at all. The deduction of the sufficiency of our condition from these hypotheses, though easy, is a little too intricate for us to give here.

We have, then, given a necessary and sufficient condition that one point, *qua* class of a-solids, should lie on the bit of line between two other points of the sort. Let us reinterpret this statement in terms of points consisting, not of a-solids, but of general convex solids. If three points, *a*, *b* and *c*, consisting of a-solids, are so arranged that *c* lies on the linear segment between *a* and *b*, and if *a′*, *l′*, and *c′* are, respectively, the points consisting of general convex solids corresponding to *a*, *b* and *c*, then it will be natural for us to say that *c′* is between *a′* and *b′* and on the line determined by them. That is, *c′* will be between *a′* and *b′* when and only when *c* contains all the a-solds common to *a* and to *b*. Now, *a* contains a given a-solid as a member when and only when *a′* contains the convex solid

from which this a-solid is formed by the adjunction of its aura, and a similar relation subsists between b and b', and between c and c'. Therefore, c contains all the a-solids common to a and b when and only when c' contains all the convex solids common to a' and to b'. Consequently c' lies on the linear segment between a' and b' when and only when c' contains all the members common to a' and to b'. Now, we have not yet defined linear segments or any such things, and this property of a', b' and c', when c' contains the common part of a' and b', is defined in purely logical terms introducing only such notions as those of part and class, involving no concrete geometrical notion, except such, of course, as are involved already in the notion of a point, which we have already defined in terms of our experience of the intersection of convex solids. We may therefore *define* a point c' to lie between two others, a' and b', when and only when c' contains the common portion of a' and b', and we shall be sure, on the one hand, that if our experience of the intersection of convex solids has the properties that are to be expected of it, this relation of betweenness will not have been misnamed, and, on the other, that this definition involves no notions other than that of our experience of the intersection of convex solids and certain general logical notions.

I wish now to define the notions of segment, end-point and line, in terms of the relation of betweenness just defined, and hence ultimately in terms of our experience of the intersection of convex solids. If a and b are distinct points, the class of all the points c which are such that c is between a and b constitutes the linear segment ab, and a and b are its end-points. The *line* ab is the class of all points belonging to linear segments which have at least two points in common with the linear segment ab. The agreement of all of these notions with the conventional

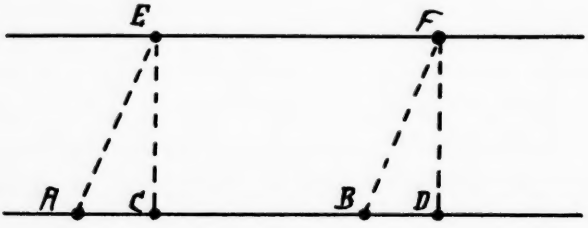
notions of segments, end-points and lines, subject to a certain reservation which we shall make in the next two lectures, will be obvious on a brief reflection. The adequacy of our definition of a line will be apparent if we reflect that any two linear segments which have two points in common are segments on the same line, while if x is any point on a line l, and s is any segment on l, a segment t can be discovered which will contain x and have at least two points in common with s.

To sum up what we have said in this lecture, we first defined a point as a class of convex solids, whereof any two are experienced to intersect, and which is further such that it contains as members all those convex solids which are experienced as intersecting all its members. We justified this definition of a point and showed that the entities which are thus defined as points are such things as one could naturally call points, providing that our experience of the intersection of convex solids has such formal properties as one would naturally attribute to it, since under this hypothesis each of our points will be a collection of all the convex solids which are experienced as containing some point, and may, since the notion of a point is only now defined for the first time, be identified with the latter point, which they are experienced as containing. We have defined a point a as between a point b and a point c when a contains the common part of b and c. From this definition alone we have derived definitions of a linear segment, of the end-points of a linear segment, and of a line. All these definitions have been made solely in terms of the experience which we have chosen as fundamental—that of the intersection of convex solids.

The work in this lecture is based on that of Dr. A. N. Whitehead and Mr. Bertrand Russell on space and time, as given in Mr. Russell's *Scientific Method in Philosophy*, Chapter IV. The definitions of betweenness and of a line are borrowed from Prof. Huntington's article in the *Mathematische Annalen* for 1912, but go back to the work of Kempe and Prof. Royce.

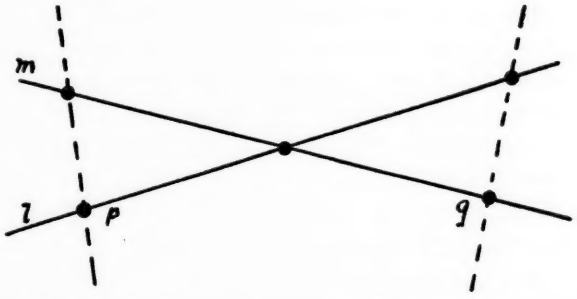
III. The Extension of Space Beyond the Bounds of Experience

In our last lecture, you will remember, we arrived at the definition of a point as a class of convex solids, and of a line as a class of points, in terms of our experience of the intersection of convex solids. These definitions, however, and, indeed, any definitions that start directly from our experience of the intersection of convex solids, must suffer from certain rather obvious defects. We intend to use our definitions of lines and of points to set up a theory of spatial measurement. To do this, we shall make much use of the construction of parallelograms: for example, we shall define the distance AB on a given line as equal to the distance CD on the same line if it is possible to construct a linear segment or piece of a straight line EF parallel to AB in such a manner that AE is parallel to BF and EC is parallel to FD. The following diagram will represent such a situation.



This demands that we are in possession of a definition of parallelism. In Euclidean geometry, to say that two lines are parallel is equivalent to saying that they lie in the same plane and do not intersect one another. We may

define two lines as being in the same plane if and only if they both have a point in common with each of a pair of intersecting lines l and m, and do not pass through the point of intersection of l and m; thus in the following diagram, p and q are in the same plane, or, as mathematicians say, are coplanar.



We might, therefore, define two lines as parallel if they are coplanar and do not intersect, without introducing any new fundamental notions into our system. But, as we have said, there are certain defects inherent in the definitions of lines and of planes that we have already given and these defects make such a definition of parallelism undesirable. These defects are due to the fact that our experience of the intersection of convex solids does not record the intersection of convex solids at the uttermost confines of space; beyond a certain extreme distance, whatever it may be, the intersection of convex solids is not experienced. As a matter of fact, I can hardly be said to have any experience of the intersection of convex solids except in the immediate neighborhood of my own body. As a consequence, any lines which would naturally be said to meet at a point lying outside the very limited region within which convex solids are experienced to intersect would, in accordance with the definition which we are considering, be parallel lines, for, by our definition of a point, a point is a collec-

tion of convex solids of which every two are experienced as intersecting, and which contains every convex solid experienced as intersecting all its members, and consequently there can be no points outside the region of the apparent intersection of convex solids, so that any two coplanar lines which fail to have a point in common within this region must fail to have any point whatsoever in common, and must, by the definition which we are considering, be parallel. Indeed, since our points are only such points as lie within a certain region of space, the lines which we defined in our last lecture are merely such parts of lines as lie within this region. Not only would this whole condition of affairs not be consonant with our natural notions of lines and of parallelism, but it would further fail to give parallelism even the most important formal properties which it possesses in ordinary geometry. For instance, it would be impossible to use the parallelogram construction as a criterion of the equality of two different linear segments on the same straight line, as a very simple geometrical construction, for it may easily be shown that our definition will make all distances along a given line equal. Hence, our definition of parallelism is at fault, and we must look around for a new one.

In what direction are we to look for this new definition of parallelism? The defect of the definition that we have just rejected—that two coplanar lines without a common point are parallel—is due to the fact that two lines which ought to have a point in common do not always have a point in common, if we define points and lines in the manner indicated in the last chapter. If we are able, then, to introduce new definitions of a point and of a line such that every two lines which ought, as ordinary, common-sense, decently-behaved lines, to have a point in common, will have a point in common, we shall have brought the problem of finding a definition of the paral-

lelism of two lines a great deal nearer solution. This new definition of a point and this new definition of a line should, if possible, be made in terms of our previous definitions of a point and a line alone, without introducing any new notions. The question we now ask is, can new definitions of a point and of a line, which make two lines contain points in common just when they ought to, be made in terms of our previous definitions of a point and of a line?

Now, we have seen that our previous definition of points gives us only the points within a certain region of space. Let us assume, for simplicity's sake—for, though this assumption is almost certainly false, it is not, as such, essential to our further argument, and it enables one to picture in his mind's eye what I have to say much better than any other hypothesis—that our points, in the sense in which points have been already defined, are *all* the points in the interior of some closed convex solid, and that our straight lines are consequently all the segments of straight lines intercepted by the surface of this solid. How are we able to recognize in terms of the points and lines inside this solid the entities, whatever they may be, that we should naturally call points outside of the solid? The problem is closely analogous to that of the recognition that a certain set of astronomical observations from various points on the surface of the earth all are observations of a single star, even though that star is utterly inaccessible to us. The problem to which I allude is not that of recognizing the star as the same star from observations at different *times;* it is the far simpler one of discovering that many observations made at the same time pertain to a single object. Let us suppose, for example, that we have four observers, a, b, c, and d, all looking at a star or planet x from different points on the surface of the earth. How

do the observers *a*, *b*, *c* and *d* know that it is a single star or planet at which they are looking?

The whole and complete answer to this question would involve considerations which are irrelevant here; it is obvious that one of the things that our four astronomers must know, however, is that they are all looking at the same *place*—that, in other words the axes of their telescopes converge on one point, or if the object at which they are all looking is sufficiently far away to be considered, for all practical optical purposes, as at an infinite distance, they must know that the axes of their telescopes point, to all intents, in one direction, or to put it otherwise, that they are all parallel. This knowledge, moreover, must be attained and is attained independently of any direct knowledge the astronomers have concerning the point to which all the axes of their telescopes converge, for this point is exceedingly remote from them, and is known by them in no other way than by these very observations concerning which we are now trying to find out why it is that the astronomers regard them as observations of a single point. When the astronomer says that at such-and-such an instant this point of space has this or that property—as for example that of being occupied by a planet—all that he means or has a right to assert must concern the observations in which the telescope is directed towards this point, for if the observations should remain the same, but the whole remainder of the universe should be changed in any manner whatsoever, the astronomer would still be entitled to make the same assertion concerning this point as formerly. The knowledge of the convergence to x of the optical axes of the telescopes at *a*, *b*, *c*, and *d* is attained by a measurement of the angles which the lines between *a*, *b*, *c*, *d*, and x make with one another and a measurement of the distances of *a*, *b*, *c*, and *d* from one another. These observations do not require any direct knowledge of x, but only of the posi-

tions of the telescopes at *a*, *b*, *c*, and *d*, for if we know the latitude and longitude of *a*, *b*, *c*, and *d*, if we know the compass-bearing of each telescope—that is, whether it is pointing east or west or southeast or north-northeast-by-north, etc.,—and if we know the slope of each of the telescopes, we know all the angles which any two lines connecting two of the points *a*, *b*, *c*, *d*, and *x* make with one another, and the mutual distances of the points *a*, *b*, *c*, and *d*. What we realy talk about, then, when we discuss the position of the planet is the aggregate of the positions of the telescopes by which it is observed.

Another thing to notice is that if we know that the telescopes at *a*, *b*, and *c* are all directed to one point, and that the telescopes at *b*, *c*, and *d*, are all directed to one point, we know that the telescopes at *a*, *b*, *c*, and *d* are all directed to one and the same point. This is rendered obvious by a simple diagram. The significance of this fact will appear if we consider that if the telescopes at *a* and at *b* are directed at one point, and the telescopes at *b* and at *c* are both directed at one point, all three telescopes need not be directed at any single point. This also is shown by a diagram.

We are now able to return to the discussion of our real problem—the problem, namely, how we are to recognize points outside that convex region of space within which all the points that we have already defined are located in terms of the points and portions of lines lying inside this region. The portions of lines lying inside our convex region—i. e., the class of all the lines that we defined in our last lecture—are the exact analogues of the telescopes or the astronomers whom we have just discussed. Just as the astronomer's statements about the position of a star really concern the positions of certain telescopes, so propositions which seem to deal with points lying beyond the bounds of our experience really concern certain collec-

tions of our lines: namely, with such as are made up of all those lines that "point at" some point lying outside our region. As far as we are concerned, such collections of lines, since they correspond uniquely to the points at which they are directed or from which they spread out, may be regarded as *constituting* these points. This situation can easily be rendered obvious by a diagram.

This definition gives rise to many problems. In the first place, how is it possible to get along in a system in which some points—those within the region of space directly accessible to experience—are the elements of which lines are classes, while other points in space ZZ those not in that region directly accessible to experience—are classes of these self-same lines? In the second place, is it possible to define the property which a class of lines has when every member of the class is directed towards some given point beyond the bounds of experience in terms of that experience which we have taken as primitive—in terms, namely, of our experience of the intersection of convex solids—without the introduction of any new experience or concept not derivable from that experience of intersection? If such a definition is possible, how are we to proceed to discover it? In the third place, how are we to tell when three or more of our new points are situated on a single line, and how are we to define such a line? These three problems will form the chief subject-matter of the remainder of this lecture and of the following lecture.

Let us take them up in the order just indicated. How, we asked, is it possible to get along in a system in which some points—those within the region directly accessible to our experience—are the elements of which lines are made up as classes, while the remainder of the points of space are classes of lines? The answer is—it is not possible, and we do not intend to try to do so in this paper. Not only is it highly inconvenient and unnatural for one

point in a system of geometry to be an aggregate of aggregates of other points, but there are good philosophical reasons—indicated by Mr. Russell in that part of the *Principia Mathematica* which deals with the Theory of Types, but too complicated and foreign to the subject-matter of this course of lectures for us to discuss here—there are good philosophical reasons, I say, for holding that no assertion which can be made significantly concerning a given entity, say x, can also be made significantly concerning a collection of collections which has some member of which x is in turn a member. Therefore, since a line, in the sense defined in our last lecture, is a class of the points which we then defined, it is impossible for one to assert any proposition significantly concerning these points, on the one hand, and also concerning the classes of lines that we intend in the future to call points, on the other. Now it is, to say the least, extremely awkward to have to phrase every proposition that concerns itself with points in one manner when it concerns itself with the points inside a given region and in an entirely different manner when it deals with the points outside this region. We shall consequently define the points inside the region directly accessible to experience as well as those outside it as classes of the lines that we defined in our last lecture, and we shall term all points *qua* classes of straight lines *generalized points*, in order that we may not confuse them with the points defined in our last lecture. Just as we agreed to regard each of the generalized points lying beyond the bounds of our direct experience as the class of all the lines which, as we should say in every-day language, are directed towards the point, so we shall agree to regard those generalized points lying within the region directly accessible to our experience as a class of all the lines which we should usually consider to pass through some point inside this region. If such a class of lines happens to be the class of all the lines which con-

tain in common a point, in the sense defined in our last lecture, then if x be the point which they all have in common and a be the class of lines, we shall say that a is the generalized point corresponding to x, but it must be clearly understood that *a is not x*.

We are now in a position to deal with our second question: is it possible to define the property which a class of lines has when every member of it, as we should usually put it, is directed towards a given point, in terms of our experience of the intersection of convex solids? It should be noticed that this is the crucial question of this entire lecture, and that our reduction of generalized points to classes of lines having some property which we should usually call "passing through a given point," but which, as a matter of fact, we wish to define without reference to any point through which the lines are supposed to pass, is in the unpleasant situation of Mahomet's coffin until we find a way of identifying this property. In the analogous instance of the astronomers and the star or planet, a collection of telescopes all pointing at a certain point in space is, as we saw, distinguished from a collection of telescopes not all pointing at any one point in space by the fact that when all the telescopes are directed towards a single point certain trigonometrical formulae connecting the latitudes, longitudes, geographical directions, and slopes of the several telescopes hold good which do not hold good when the telescopes are not all aimed at any one point. Such a method of determining whether or not all the lines of a given collection are directed to a single point is inapplicable to the case where we are to define the generalized points of space solely in terms of its points and lines in the sense of our last lecture, for we have as yet no definition of an angle or of a distance nor of a slope: all that we have defined up to this point is the set of all the points and linear segments that lie within a given region. Our problem

hence reduces itself to that of the determination of such classes of lines as are made up from all the lines that pass through a given region—the region, namely, within which we experience the intersection of convex solids—and some chosen point inside or outside of this region, in terms of the intersection-relations of those portions of lines lying inside the region.

We can, however, narrow our problem still further, and indicate the method of its solution with still greater definiteness, if we remember a certain fact about straight lines which we pointed out when we were discussing the case where several observers are looking at a single star. It will be remembered that we showed that if the axes of the telescopes *a*, *b*, and *c* converge to a single point and the axes of the telescopes *b*, *c*, and *d* likewise converge to a single point then the axes of all the four telescopes *a*, *b*, *c*, and *d* all converge to the *same* point. As we may easily show by a diagram, we may generalize this statement and say that, given any collection of telescopes, if there are two among them, say *a* and *b*, such that if *x* be any member of the collection of telescopes the axes of *a*, *b*, and *x* all converge to one point, then all the axes of the telescopes of the collection converge to a single point. The converse of this statement is even more obviously true. We can thus define a collection of telescopes as one, the axes of all of whose members converge to some one point provided that it contains two members the axes of which intersect and that the collection is made up of all telescopes which form, taken together with these two, a triad of telescopes whose axes converge to a single point, and in addition of these two telescopes themselves. If, that is, we are in possession of a criterion of the convergence of a triad of telescope-axes, we are able to define the collections of all telescope-axes converging to some point or other. In exactly the same manner we are able to define certain classes of the

lines we discussed in our last lecture as classes of all the lines which, we should ordinarily say, pass through some point or other, whether that point is or is not within that region which is directly accessible to our experience, or in other words, as generalized points. If we have a criterion which enables us to discover when any three lines converge to any point whatsoever in space, for the property of line-triads which reads, "If two line-triads each of which is made up of three lines converging to a point possess two lines in common, all four lines making up the two triads pass through some single point," is not confined in its application to the axes of telescopes but applies equally well to all kinds of lines. Therefore, if we are already in the possession of a definition of a convergent triad of lines, we may define a generalized point as the class of all lines, in the sense in which we defined lines in our last lecture, which either are one of two given lines, say l and m, or form together with l and m triads in which the three members of the triad stand to one another in the relation which is ordinarily denominated 'all passing through the same point,' provided only that l and m are distinct intersecting lines—that is, distinct lines which form two of the members of some triad of lines which all would naturally be said to pass through some point. If, then, we are able to give a definition of the relation among three portions of lines lying inside a given region of space which we should naturally call, that of all being directed towards some one point and which the mathematician terms the relation of concurrence, which involves only such notions as we can define in terms of the points and lines of our last lecture, we are in a position to define the class of all generalized points in space, wherever they may be situated. One of the notions which it is permitted for us to use in the definition of the concurrence of three lines is that of the relation which two of the lines of our last lecture bear to one another

when they possess in common one of the points of our last lecture as a member, for this notion can be defined in terms of points, lines, and notions of pure logic alone, and consequently ultimately in terms of the experience of the intersection of convex solids and of in addition only such notions as belong to pure logic and not to concrete experience.

This demand will be satisfied if we give an adequate definition of the concurrence of any three lines which do not all lie in a single plane, for we can define in terms of this relation the concurrence of any three lines whatever, whether they are concurrent or not, in the following manner: the lines *a, b,* and *c* are said to be concurrent if *d* and *e* are two lines such that each of the three lines *a, b,* and *c* forms with *d* and *e* a triad whose members are concurrent *and do not lie in the same plane.* Now, it is an extremely easy task to give a definition of a relation between three lines not all in the same plane, which, though it is slightly more general than the relation of concurrence, includes the latter as a special case, and is only *slightly* more general than it. This new relation is that which holds among three coplanar lines when they are either all concurrent or all parallel. We shall introduce into the definition of this no notion which we have not already defined in terms of the experience that we have taken as fundamental—the experience, namely, of the intersection of convex solids. We can define a plane, readily enough, as the class of all those points, in the sense in which we have already defined points, which lie on any line which has two distinct points in common with some given pair of lines that themselves have a point in common, but do not coincide. As the lines that we have already defined really represent those segments of the lines of ordinary geometry intercepted by the surface of that region of space within which convex solids appear to intersect, our planes, as they are now defined, will actu-

ally represent the planar areas intercepted by the surface of this region, provided that it is possible to draw from any point of such an area a line cutting any two given linear segments intercepted by the surface of the area in two distinct points. That this is possible we may readily show to be the case under the hypothesis, which we have every reason to believe satisfied, that the region of space accessible to our experience is convex. Now, it is a familiar theorem of elementary solid geometry that if p, q, and r be any three distinct planes of which no two are parallel and which do not all possess any line in common, then the intersection of p and q, the intersection of q and r, and the intersection of r and p will form a triad of concurrent or parallel lines. The proof of this theorem is simple, and the situation it represents is illustrated by the corner of a room, where the walls and floor represent p, q, and r, and the three edges of the room that meet at the corner are the lines of intersection of pairs of the planes p, q, and r. The case where the three lines are parallel is represented by the three faces and the three edges of a triangular prism. From these examples, it is further easy to guess the truth of the converse theorem of that which we have just stated: three lines not all in one plane are concurrent or parallel *only* when they are the three lines of intersection of pairs of the planes belonging to a certain triad. If we apply these theorems to the lines of our last lecture, under the hypothesis that these represent the linear segments intercepted by the surface of a certain convex region, we shall obtain the result that three of the lines of our last lecture that do not all lie in one plane are concurrent, wherever the point of their concurrence may be situated, or parallel *when and only when they are coplanar by pairs.* Since the relation of coplanarity among the lines of our last lecture has been already defined and the concurrence or parallelism of three lines not all coplanar has not yet been defined in terms of

our experience of the intersection of convex solids, we may regard the equivalence expressed in our last sentence as a *definition* of the concurrence or parallelism of three lines not all coplanar.

We may now go on and say that three of our lines, l, m, and n are concurrent or parallel, whether they are all coplanar or not, when and only when they are all distinct and there are two of our lines, a and b, let us say, which are such that a, b, and l, a, b, and m, and a, b, and n form three triads, respectively, each made up of three concurrent or parallel lines that are not all coplanar, in the manner that we just defined in the last paragraph. This definition *resembles* the definition of the concurrence of three lines, whether they are all in the same plane or not, which we suggested earlier in this lecture, but it differs from the latter in that it defines the *concurrence or parallelism* of any three lines, and not their simple concurrence. To prove the adequacy of this definition is a simple matter, and reduces itself to the proof that if a and b intersect, and l, m, and n each form with a and b a triad of concurrent lines, l, m, and n are concurrent, and that further if a and b are parallel, and l, m, and n each form with a and b a triad of parallel lines, then l, m, and n are all parallel to one another. These two propositions are obvious on inspection. We thus see that our definition of parallel or concurrent triads of lines covers those triads, and only those triads, of the lines of our last lecture that we should naturally call concurrent or parallel triads.

To sum up what we have said in this lecture: we saw that the definitions of our last lecture yield us only those points and linear segments within a certain limited region of space. We found it necessary, therefore, to search for a definition of all the points and lines of space in terms of those lying inside this region, and found our problem analogous to that of the astronomer in the location of a

planet by its parallax. We learned from this example that if we were in possession of a definition of the concurrence of three lines, we could define a point anew as a class of concurrent lines of the sort defined in our last lecture, and thus obtain a system of points extending throughout space. We searched for such a definition of the concurrence of three lines, but found instead a definition of the concurrence or parallelism of three of the lines of our last lecture, involving no concrete notion other than that of our experience of the intersection of convex solids. The problems that remain before us in the next lecture are first, that of observing what effects the difference between the relation of concurrence, for which we sought, and that of concurrence or parallelism, which we obtained, will involve with respect to our new points and their definition, and secondly, that of the definition of the lines that connect our new points.

NORBERT WIENER.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY.

RELIGION AND THEOLOGY

RELIGION is still confused with theology, although the latter is only one aspect of the religious attitude and explanatory theory. Religion, psychologically speaking, is an attitude. Philosophically or sociologically, it is an explanation, or a justification, of something which the subscriber wishes greatly to have explained or justified. Its social function is that of the control of the world in which its devotees live, or of those parts of their environment with which they are most vitally concerned. These two aspects, the affective attitude and the philosophic explanation, are indispensable to all religious experience and belief and are found together in all religious doctrine and dogma, just as creed and the reverential attitude are always associated. If the thing to be explained is a mystery which has greatly puzzled men, or which they profess not to be able to comprehend by means of the reason, or which they cannot easily justify from current causal and moral philosophies, the explanation and justification appear all the more wonderful to the believers and are greatly prized. In such cases the attitudinal side is highly developed, the believer's emotions often rising to the pitch of adoration or awe. A religion which makes such a strong appeal as to become practically exclusively accepted within the region to which it has spread is likely to be based philosophically on a very great and absorbing mystery, on the one hand, and to make a tremendous appeal to the emotions in the

forms of reverence, gratitude, love and adoration, especially for the personal objects or factors involved in the explanatory theory, on the other. Thus, the great historic religions have usually been concerned with the explanation of the whence and whither of man, a solution of his destiny as cast along the simpler and more cosmic lines, while often neglecting the terrestrial and complicated immediate aspects of his destiny, regarding them merely as incident to the other more cosmical processes. The personalities conceived of as directing this destiny have been the objects of the strongest attitudes of love and hate, gratitude and fear, and other emotions, according as the dispensation of destiny was regarded as favorable or unfavorable to the recipient.

A theological religion is one in which personalities, especially gods and goddesses, play leading roles. The mystery or problem is there, as in all religion, but the explanation is closely tied up with the personalities of the beings who are supposed to dominate the cosmic or terrestrial situation. In fact, since man at first knew neither physics nor chemistry, biology nor psychology and sociology, but explained all phenomena in terms of personalities or personal causation, in the early stages of religion the gods themselves were the explanations. Primitive, and even barbarian and early civilized man did not often seek to go beyond personality into the organization of forces and factors behind these. In fact, he reversed the scientific method of explaining social phenomena, regarding the personality in the situation as the cause of other phenomena rather than as the product of them. Environment was constituted for him practically entirely of personality or quasi-personalities and not of the elaborate complexity of physical, biological and psycho-social factors and combinations of these, as we are now learning to conceive it. All early religions were therefore theologies or pre-theolo-

gies, in which causation and process were thought of as personal or by analogy of personality activity.

Only with the coming of a more objective method of analysis of natural and social forces has religion come to be divested increasingly of the theological and to take on the metaphysical and scientific aspects and forms. The scientist of today is inclined to explain the theological personalities as ideal objectifications of man's desires and sense of the right and proper organization and balance of things, always stated in terms of personal relationship and causation, because he knew no other formula. He clothed his divinities in the perfect forms of those attributes which he himself possessed in imperfect measure. What he should have desired to be, the gods were—omniscent, omnipotent, immortal, sometimes all-good, always all-cunning, omnipresent, with infinite powers of transmutability and of visibility and invisibility at will. They possessed his own emotions and form, or those of the animals he loved, feared, or respected, but always in greater perfection than he had them, or at least in more perfect adaptation to the purposes or ideas which they were supposed to serve. Thus the gods have been regarded as the anthropomorphic creations of idealistic and striving, self-conscious, and more or less socially conscious, man.

As man's analysis of his external world proceeded to the point at which he began to be able to formulate abstract statements of physical forces, to see climatic, geographical and biological factors at work, his tendency to state all phenomena in terms of superior personality manipulation, by analogy of his own immediate experiences, slackened. As the prototypes of physics and chemistry appeared and the concept of natural law developed theological personalities began to fade. A metaphysics began to take the place of theology in the minds of the learned, and this was in turn succeeded by a developing body of scientific

concepts. Causation became increasingly abstract and less and less personal. The gods were reduced, in the thinking of the philosophers, from the role of directors of the cosmical and human social processes to that of being the agents of natural law, which was now supreme.[1] Among the scientists of today there is some danger that theological personalities may become merely mythological, disappearing altogether from their thinking as active causal concepts.[2] The scientific and logical methods of abstraction have apparently triumphed in the world of the intellectuals, if not in the minds of the masses, and physics and chemistry and bio-physics and bio-chemistry and the mechanics of the physico-geographic and the psycho-social environments have become regnant as explanatory concepts instead.

But religion has not ceased to exist with the decline of the gods and the theologies which explain them. Although the popular tendency is to regard religion as exclusively theological and to disregard any non-theological definition of religion, the movement above described, away from personality over to abstract explanation, may be observed in all the traditional religions which have adherents in the western civilized nations. The tendency is particularly to be observed in the various forms of Christianity and Judaism. We have not lost our interest in the same old mysteries of life and mind and matter, the whence and the whither, the origin and the destiny of man; but we have learned to explain these mysteries and to justify the processes—evolutionary or otherwise—more and more in terms of scientific processes and concepts. Also we have become increasingly interested in the present aspects of man's destiny, with the result that religions have become less cosmical and more social at the same time that they

[1] The philosopher Grotius made the statement that God could not act contrary to natural law, but must act in conformity with it.

[2] See Leuba, J. H., *The Belief in God and Immortality.*

have become more scientific. The concept of heaven and the heavenly society becomes less definite and that of a better human society on this earth grows to take its place. Social ethics tends to grow at the expense of ritualistic observance based on a theological past. Some have even prophesied that the great historic religions, subjected to the transforming influence of modern scientific concepts, must either lose their theological character and become humanized and socialized, or perish, giving place to newer religions which embody the explanatory theories and the idealistic devotion to them which conform to our own age.

The evolution of the objects of worship shows very clearly the growth in religion from a theological to a scientific content. There seems to be good psychological argument for Professor Giddings' contention that the first object of worship in nature was not fetisch, spirit or god, but the "Great Dreadful." Early man, just emerging from the prehuman existence, or even throughout the stages of savagery, could not have defined personality in any very tangible terms. He felt it rather than described it. Other men must have seemed rather vague and indeterminate to him, as indeed did his own personality or self. We who are accustomed to rather sophisticated analyses of self, employing as aids to the process a considerable equipment of psychological terminology, comparatively easily distinguish ourselves from others on the sensory side, at least in adulthood. But even we confuse others with ourselves when we undertake to interpret them, constantly reading our own personality traits, attitudes and beliefs into them. Primitive man, possessing few verbal aids to discrimination, must, as the evidence indicates, have had infinitely more difficulty. The very young child suffers from the same limitations in technique, only gradually learning through many trials to distinguish person from person and others from himself. Both the child and the

primitive man fear the dark and those aspects of nature which they have not learned to account for in such terminology and through such processes of analysis as will remove the mystery from them. If the civilized child does not fear the "Great Dreadful," or nature, when he grows up, as does the primitive man, and as he himself fears the dark, it is because he has by that time acquired a method of accounting for things in analytical terminology which the primitive man did not possess, and also because his economic and technic position generally in the world has so greatly improved over that of the primitive man's that nature is to him more beneficent and less voracious and terrifying.

Such a vague characterization of external things, lumping them all more or less together and regarding them as fearful or harm-doing, is a sort of beginning of personality analysis. Closely allied to it is the singling out of certain objects, which are supposed to possess general or specific powers to produce certain results or ends. These objects, in a more sophisticated stage of development, are called fetishes. They are used as means to the control of some aspects of the environment. While such objects have not personality in the sense in which human beings possess it, their powers are evidently interpreted or imputed on the analogy of the observed or imputed powers of human beings. There is no other method of interpreting them known to the primitive man. Furthermore, they are supposed to possess not only powers, but also attitudes, which change from time to time and which can be modified by certain ascertained procedures, in much the same way as human attitudes are ascertained and modified. Such objects-with-power are not yet abstracted into the category of physical objects, that is, divested of personality. That comes with insight and completeness at a later stage. Also, such objects-with-power partake of the same vagueness

and instability from which all personality suffers in the stages before there is a technique for its analysis and a terminology for its description. But such a method of explaining magical or non-human control over the environment does represent a real advance in the definition of personality over the preceding method upon which it is superimposed and which it begins to supersede. It represents a distinct advance in definition and localization of causation and activity traits, even though these may be merely imputed and hypothetical ones. There is less vagueness and generality, more that can be definitely accounted for, in the causal and control process with such an explanation. We can imagine that the primitive man who had reduced his world control process from the "Great Dreadful" to fetishism must have felt more secure and comfortable than the one who had not done so, because he could "put his finger on things," as it were. And one of the functions of religion is to make the world a comfortable place for its inhabitants to live in.

The spirit, as an instrument of magical control over the environment, is but an extension of the principle of the object-with-power. It represents the beginning of the tendency to distinguish between physical or inanimate objects and the animate objects or objects with personality. The spirit tends to associate itself with some inanimate object which it uses as a dwelling place or refuge and which it controls. This concept of a distinction between a sheltering object and the sheltered spirit was undoubtedly helped out by the dream of experience of early man. He believed that his spirit had the power to leave his body, which remained quiescent in the place where it had gone to sleep, and go off about its business. It might go hunting, travel across the mountains, visit the departed dead in the spirit world, engage in warfare, or undertake many other activities. This observed dualism of his own nature might easily be

imputed to other objects also, on the accepted basis of regarding them by analogy with human or personality beings. The whole process of imputation is, of course, quite generally familiar. The spirit of the inanimate object seems to be much more freely dissociated from its "shelter" than that of the human being, possibly because the former is much less active. Also this object may be used as a shelter for many different spirits, although the spirits seem to prefer their regular and habitual dwelling places. Human bodies may also harbor alien spirits, although normally they serve as the homes of one particular spirit.

With this development of the spirit as the embodiment of the personality, the definition of personality is greatly extended in detail and completeness. With the multiplication of spirits, especially of types of spirits, the distinctions of personality become easier to perceive and enumerate. The spirit itself is, of course, at first but poorly developed in personality attributes. It is little more than the object-with-power, the power having become detachable as spirit. But the spirits grow in richness of content as the number of things they can do multiplies. From the spirits in their well-developed stages we make the transition to the gods. A god is a spirit embodied in human or animal form, although not necessarily in flesh and blood. The god usually possesses the power of making itself invisible when it desires, of transmutation into other forms than its habitual one, and of very rapid, if not instantaneous, movement. It also, of course, possesses the power of magic, or of acting by fiat, in a very high degree. The god is the almost perfect embodiment of magical power. Gods, like spirits, are domiciled, at least in the earlier stages of their development. They have their favorite dwelling places and places of dalliance, and in these places they may be found ordinarily. But their range of movement is very great. The god is a much more complete person-

ality than the spirit. By the time the god has developed out of the spirit, human personality analysis and characterization have also developed greatly. They have approached in definiteness and richness somewhat to the condition we have today, although there is, of course, no definite psychological technique for that analysis of which we are capable. This high degree of human personality analysis, carried over by imputation to the spirits, made it possible for a rich galaxy of gods to develop in the image of man.

This is the stage of polytheism in the worship of religious objects, and strictly speaking it is the first full-fledged stage of theology. But in reality there is no essential difference between the stages of religious development in which the gods play a part and those earlier ones in which spirits and objects-with-power are the agencies of magical control, except the differences in degree of personality development. The essential distinguishing characteristic of the theological stage of worship is personality, and there is a degree of personality in all of these types of objects from the "Great Dreadful" to the gods themselves. Only in the later stages of religious development does personality begin to give way before the abstract natural forces which tend to resolve themselves into physics, chemistry and bio-physics and bio-chemistry and the various permutations and functional organizations of these.

The gods of the polytheistic stage are enriched personalizations of the spirits, their content growing with the growth of personality in general. But they are, of course, always personifications, and in the main of two types of objects, the natural physical forces and attributes and of the human emotions and traits. Thus, in the Greek theology, Jupiter is the thunderer, the god of lightening, of rain, of the majesty of the heavens. Vulcan is the personification of the volcano and the underground fires, a black-

smith at his forge, and a worker in metals. Neptune is the god of the sea, of the rolling, galloping waves; Ceres of productiveness; Apollo of the sun in the heavens, the torch bearer of the day. The passions are equally well represented. Venus is goddess of the tender passion, of love and voluptuousness and of beauty, as soft and white as the foam of the sea from which she sprang. Mars, whom even the ancients made the companion of sexual love, is the thundering god of war, cruel, fierce, powerful, delighting in the torch, more effective in the fight than powerful in intellect. The muses, the fates, and numerous other divinities, stand out as the representatives and patrons of the arts, or of some common or crucial experience in life. Early peoples, not having abstract symbols, with which to express their interpretations of human nature and social processes or even of physical nature and processes, symbolized them concretely in personifications, finding as far as possible, analogies in human action and human personality with which to express their ideas and attitudes.

Of course, we must not expect to find the polytheistic pantheons made up of merely simple personalities representing simple traits when we discover them formed at the threshold of history. Nor should we think of each god or goddess as always having his or her own well-marked-off sphere. There are, as a matter of fact, all sorts of syntheses and overlappings, conflicting jurisdictions and multiple and divided personalities, so to speak, among the gods. Diana is at least a two-in-one goddess, representing both the chase and chastity, and also closely allied to that subtle and dangerous goddess, Luna. Minerva is not only the goddess of wisdom, but she is the titular defender of the Acropolis, where she stands Mars-like with sword in hand and her shield upon her arm. Mars is both fighter and master-of-lust. Jupiter seems to have absorbed half

a dozen personalities, more or less. This multiplication of personality within the same divinity probably arose in the main from syntheses and absorptions of different gods into one. Thus, a group of divinities inhabiting the same region or representing different attributes of the same general aspect of nature or human character were apparently consolidated into one powerful deity. This latter procedure was apparently the one which gave rise to the composite character of Jupiter. Perhaps the procedure of synthesis explains the two diverse characteristics of Minerva. In other cases still, gods were imported and assimilated to those they most resembled in the adopted or adopting country. In this way a divinity, at first fairly simple, might become quite complex and even contradictory in his personality. Another source still of this complexity of character of the gods was undoubtedly a natural growth due to the gradual expansion and multiplication of personality traits in the worshippers and the accumulation of legends about the persons of the gods, building into them the traits and values which the stories attributed to them. If the gods were social fictions, equally truly may it be said that they were in large part the product of the habit of men in their leisure moments of indulging in fiction as a method of amusing themselves.

This tendency toward the synthetic fusion or synthesis of the gods proceeds until monotheism is approached, in which, theoretically at least, the valuable elements of personality and attributes of power of all the gods are merged in one supreme and all-powerful and all-wise divinity. No people as a whole ever accomplished this fusion entirely, although it is possible that some sects within a people have attained to the complete concept of monotheism and the worship of a single god. What usually results is the emergence of a powerful and dominating synthetic personality in a class by himself, with a number of subsidiary

tiers in the divine hierarchy, such as the devil (who, indeed, sometimes appears successfully to defy God), the archangels and angels, good and evil spirits, saints, etc. The highest ideal conceptions of the most approved personality values are attributed to this supreme god, in addition to perfection of magical or fiat powers, omniscience, omnipresence, power of invisibility, of annihilating space and time, transmutability, and all the other attributes which belong to the conceptions of magic rather than of science —truly what each individual would like to possess for himself.

One of the greatest aids to the development of monotheism is undoubtedly the achievement of social unity among a people. As they progress from the tribal to the national stage of economic and social development, as their cultures become merged and amalgamated, when they can think largely with a single mind, the diversity and multiplicity of gods tend to disappear and the concept of divinity acquires a unity which is comparable to that of the national mind. The monotheistic conception of divinity having once been established with a fair degree of perfection, it in turn serves as a focus for the drawing together of divergent strands of thought and ideals in the nation. Especially, if the concept of the monotheistic divinity is a dogmatic and coercive one, it is likely to attach itself to all forms of psychic expression and to draw them into close conformity, thus in the end developing a highly self-conscious and characteristic culture. Such was that of the ancient Jews, and such is that of all countries which have a dogmatic national religion, whether it be theological or non-theological. The dominant German religion in the last forty or fifty years was only incidentally theological, but was primarily that of national aggrandizement—a politico-socio-economic religion—and it exhibited as many signs of intolerance and coercion in behalf of its

"Kultur" as did the theological or monotheistic religion of the Jews and other peoples in like stages of development.

Pantheism is essentially depersonalized monotheism. All the ideal attributes of the personal god remains, such as omnipresence, omnipotence, and the rest, but they are reduced to essence instead of being super-human qualities. The god has now passed out of the human or anthropomorphic stage and has become a divinity of principle. Pantheism represents the coming of the metaphysical stage of religion, and since it grows out of the theological stage it carries over some of the theological attributes into the metaphysical stage. Pantheism appeared first among the philosophers, those who had begun to speculate about the nature of matter and force and motion and the qualities of objects and had learned to trace them back of the personality concept. It came when the analysis of physical nature reached the point where fiat and will gave way to principle or the concept of uniformity in causation. This concept of uniformity is the basis of metaphysics and a metaphysical interpretation of the world is the basis of pantheism. This is also the basis of the scientific method, but the metaphysical interpretation falls short of the scientific, because it lodges the cause of phenomena in certain essence qualities in the objects themselves. It subjects motion and force to a sort of mechanical analysis, but leaves the mystery of the theological personality in the thing, although it is transformed in nature within the object and uniformized as among objects. Thus, uniformity and inevitability are substituted for the fiat will and whimsical unaccountability in the physical world. This substitution was inevitable, once the phenomena of the physical world had been sufficiently analyzed to establish the regularity of their occurrence. Metaphysics banishes spirits and gods or merely makes them ornamental or, at most, a great *deus ex machina*, which sets the thing in

motion and then does not interfere with it again. As Grotius said, God cannot act contrary to natural law. So pantheism neglects God and substitutes Natural Law, which now becomes a depersonalized entity or cause, with all the attributes of divinity except personality, and which pervades the universe as an essence inhering in all space and matter. Modern vitalism is a belated form of this metaphysical concession to the theological persistence in the thinking of man about the events surrounding him.

Pantheism began as the religion of the philosophers who had proceeded beyond the theologico-personality concepts of causation in their analysis of physical nature and the universe. For a very long period of time it did not get into the popular thinking at all, for the masses of the people did not advance beyond theological and personality concepts in their causal thinking, at least with respect to the more ultimate phenomena of the world and universe. However, it may be noted that they began to think in quasi-mechanistic terminology regarding purely local and immediate phenomena before they did of the more distant and less easily observable phenomena. But with the spread of physical analysis and of mechanistic concepts, which has become rather marked in recent times, the masses of the people have absorbed enough of this method of thinking to arrive at a sort of metaphysics of matter and of life and to depersonalize their concept of God. The growth of astronomy, as well as of the other physical sciences, has contributed largely to this. The constant searching of space and the breaking up of matter into its elements has left no place for God as person, in any nearby part of the universe, at least. But there may still be room for God, with His old immediacy and other attributes, as essence, pervading all space and matter. The average man does not think out his change of view with so much logical clarity as this statement would seem to imply, but this is about

the sort of conclusions he arrives at, working more or less subconciously and incoherently.[3]

If the pantheism of the theists never did become the universal belief of the masses, much less has the a-theism of the scientific interpretationists in religion. This method of answering the question as to the nature and meaning of the universe was developed primarily in the nineteenth century after the rise of modern physical science. For the mysterious essences in nature and matter were substituted, through a process of scientific analysis and synthesis, categories of mass, motion and energy. These were expressed in definite quantitative mathematical, instead of vague mystical, terms. As a result of this substitution the laws and principles of science were made to replace the old concept of natural law, and science grew at the expense of metaphysics. Since the type of religion always follows and conforms to the method of interpreting phenomena, there now succeeded, in the thinking of the scientific philosophers, to the old pantheism which had grown out of a metaphysical interpretation of phenomena, a scientific interpretation of the universe and of life, couched in terms of scientific law and principle and formulae.

This new scientific interpretation of the universe was given by the theological and metaphysical interpretationists in religion the term "atheism," sometimes varied into "agnostic" and "infidel," to suit the mood or bias of the apologist or critic. Indeed, it was atheistic, since, as LaPlace said to Napoleon concerning his *Celestial Mechanics,* its treatment had no need of the hypothesis of God or the gods. Personality and personal causation in cosmic matters, and in the larger and more abiding aspects of

[3] In testing out my elementary students, mainly freshmen and sophomores, as to their concept of God, I was surprised to find that the majority of them did not think of an anthropomorphic god, at least with regard to form, at all, but had the concept of an all-pervading essence. This is indeed a great change in the popular attitude since the time when the Dorè engravings represented God as a robust, benign patriarch of fifty or sixty in human form.

terrestrial affairs, drop out and mathematical statements of the correlations of matter, motion and energy take their place. The sphere of personality causation is narrowed down to human and social phenomena; and there begin to be those who predict that a further analysis of phenomena and the development of scientific method will remove even these spheres from a pure personality interpretation.[4] Herbert Spencer was one of the first to grasp fully and set forth systematically this new viewpoint of interpretation. Following the publication of his *First Principles* in 1862 many contributions were made in this same field of philosophic interpretation and the controversy between this viewpoint and that of the theologians and the metaphysicians waxed warm for more than half a century. Comte and the Positivists had approached this viewpoint in a more metaphysical and somewhat sentimental religious scheme even before Spencer's essay appeared, and the idea had been gaining volume and clarity since the time of the French enlightenment of the eighteenth century.

The function of religion, whether theological, metaphysical or scientific in its method of interpretation, has always been to control the environment directly or to provide a theory of environmental control. The great historical religions, which have absorbed the effective attention of the masses of mankind during historic times, have endeavored to expand this theory of control and adjustment to cover the whole of space and time and to look upon man and his social organization as the product, although chief terrestrial objective, of this cosmic process. The chief questions asked and for which answers were sought in the nature of the cosmic religious order was the whence and whither of man and his duties relative thereto and to the author or authors of this destiny while

[4] For a discussion of this subject, see Ellwood, *"Objectivism in Sociology," American Journal of Sociology*, XXII, 289-307; and Bernard, *"The Objective Viewpoint in Sociology," Ibid*, XXV, 298-325.

he sojourned on the earth. The different great religions made different answers to these questions, but none was uninterested in them. Before the great historic religions developed, the problems of control were more immediate and local and man's religious interpretations less philosophical. This is the period in which control by what is called magic was dominant, but the method of magic continues still to be, at least in the larger affairs of man—those which seem to be out of his immediate reach and purview—the professed method of control, even in the professedly theological stages of religious development, that is, in the polytheistic and the monotheistic stages, and to a large degree in the pantheistic stage. Magic disappears, or tends to disappear, as the method of control, only as man's analysis and abstraction of the physical forces and processes of nature give him a capacity for refined and progressive adjustment to his environment, which he lacked in the stages of the dominance of magic.

When the system of religious control reaches the stage of development at which it becomes a theory of control instead of merely a technique of control, it serves as a source from which the individual may draw the materials for a philosophy of life for his guidance, although the average individual is likely to be guided in his conduct more by his subconscious valuations and impressions than by his consciously arrived-at determinations. A system of social ethics, at first mainly ritualistic and later more or less rational, grows up within this general system of religious philosophy to take care of the every-day human contacts. As the scientific aspect of religion develops at the expense of the theological or magical, the philosophic explanatory system develops into a correlation of the sciences, and the social ethics is attached more firmly to the sciences contributing to this philosophic system and is transformed into a sort of social philosophy. Also as the

content or objects of attention in religion are transformed from personalities or divinities into scientific principles or formulae and systems, the strong emotional attitude connected with the former is likely to be weakened before the progressive intellectualization of the latter. Dogmatism, so often the accompaniment of theologies, is much mitigated in the scientific religions; change of idea and content is accomplished much more easily, with the result that explanations of environment are brought and kept much more nearly up-to-date.

So noticeable is all this that many people would not speak of religion after the metaphysical stage is reached, at least as a general system of explanatory philosophy, but would prefer to give it simply the name of philosophy or science. To them there does not appear to be enough of the affective element to justify the term religion. But there is always a strong affective attitude towards the content of any general philosophic or scientific explanatory system which is used to orient one's thinking, or action, toward the major problems of life. People are not lacking in devotion to that general synthesis which they often call just "science" and which they use for such guidance. "Science" has its perfervid partisans as well as "religion," or, better, as theological religion or some metaphysical cult. If there is not as strong dogmatism about science it is primarily because the objects of affective attention are abstractions instead of concrete idealized personalities and they are arrived at by intellectual analysis instead of emotional synthesis.

Ex-president Eliot was quoted some years ago as stating that the religion of the future would be the outgrowth of and moralization of modern science. Many people have already developed science into a religion in this way. The age-old question of "whence" is answered for them in the facts of geology and biology, including the data and prin-

ciples of embryology, heredity, etc. They resolve the problem of "whither" by appealing to the facts of chemistry and the related sciences. Their problems of present adjustment to and control of their terrestrial and social life are cared for through the various mental and social sciences. They work up for themselves a synthetic collection of the principles and data in these various sciences which best meet their needs of explanation and adaptation and control and constitute them a more or less absolute category of truth, for which various degrees of reverence are felt and expressed. Some of the ethical culture societies professedly base their principles and teachings upon such an organization of scientific principles. Beyond a few such sporadic manifestations, the "religion of science" has not established a "church," unless the institutions for research and the dissemination of scientific knowledge could be called such. These means for the popularization of scientific truth, especially in its relation to human life and society—and all religion centers around human and social uses, at least in its affective content—will become increasingly common.

When we turn to the secondary religions, those that group themselves around some particular problem of adjustment or control, rather than the great systems of explanatory religious philosophy, we find these same principles and tendencies exemplified. The old belief that human welfare can be fostered by magic is mainly disappearing and the idea that God interferes in the process of events to promote the individual's success here or to prevent disaster elsewhere is also no longer generally held, at least by the better educated people. It may be said, therefore, that the theological interpretation of the secondary human adjustments, the theories of social and community welfare and of individual perfectability, are no longer dominantly theological. They are usually meta-

physical or scientific. As examples of the metaphysical secondary religions may be cited the various "New Thought" theories of individual psychic adjustment and Christian Science, both of which have attained much vogue and even threaten the dominance of the old theological religions. The problem or "mystery" in both these cases is how the individual can achieve an efficient functional adjustment to life. The solution to which the adherents are devoted, which constitutes the body of their religious belief or doctrine, is found in the principles of auto-suggestion and the teachings of Christian Science. The solution is primarily subjective, these religions not being social religions as such are ordinarily understood. They substitute for the old theological subjective adjustment made through prayer a new one of auto-suggestion and a modified form of psycho-therapy. Their explanations and theories are mainly metaphysical, but they utilize much scientific data and technique.

Other secondary religions which have come more fully under the control of scientific method, and which at the same time show a trend more largely from the subjective and individualistic motivation over to the social, are such social programs and cults as woman's suffrage, the conservation of natural resources for the future improvement of civilization, single tax, socialism, anarchism, classicism, cubism, and the like. In each case there is a problem of adjustment or control which seems a vital, perhaps the most vital, problem to those who are attempting to solve it. The proposed solution which they accept constitutes a religious doctrine, although it may be also a more or less approved or exploded theory in social science. Thus, the belief in monarchy or in socialism, to take examples at random, as the proper methods of organizing and controling human society for certain expressed ends becomes a religion in the minds of its convinced and enthusiastic

supporters. If one cares to look for analogies to the great historic religions they are not lacking. In the case of socialism the problem which presents itself to its enthusiasts is the establishment on earth of a social order which will bring justice and happiness to every one. The body of doctrines or beliefs consists of the economico-social theories of state ownership of the general and large scale service utilities, regulated distribution, surplus value, etc. The ultimate goal, which may perhaps be compared to the Christian heaven, is the "Co-operative Commonwealth," which formerly was a true religious slogan among the socialists. The socialist bible, among the orthodox, is or was until recently Karl Marx' *Das Kapital.* Spargo tells of a poor working man who every Sunday walked seven miles, each way, across the city to spend the day with a friend, religiously spelling out the words and sentences of this work, most of which he did not understand.[5] At one time the socialist societies undertook to edit and introduce an extensive Sunday School literature based on socialist works, and communist groups have been accustomed to meet regularly Sunday mornings to read and expound the writings of Lenine and his associates.

What is true of socialism is in greater or less degree true of all the modern issues and programs. They take on a religious significance. Social work becomes to many a true religion to which devout workers consecrate their lives. The Salvation Army is an example of a mixture of the humanitarian motive of salvaging human derelicts and traditional Christian dogma. Here a new secondary social religion is fused with a great historical theological religion. But in the social work of the associated charities and the various welfare and protective leagues the theological element is almost wholly replaced by the principles and technique of social science. No theological doctrines are taught

[5] See Spargo, *The Marx He Knew,* for a good example of the religious attitude in Socialism.

by the secular societies of this type and none is insisted upon in the recipient of relief or protection or guidance. The object is social and individual improvement through a better adjustment of the individual to the social organism. Thus the principles of the social sciences, and of all science in fact, become the theoretical or explanatory content of the new scientific secondary religions, just as they tend to be for the newly evolving great explanatory philosophic system which many men are building up to supplement or replace the historic systems of religious explanatory philosophy.

The affective element may be much stronger in the secondary religions than in the great primary explanatory religious systems, especially when based on modern science. Relationships and concepts are likely to be less abstract in the secondary religions. Control of environment tends to be more directly control of human personality. The human ends, the results for concrete living, for enjoyment and suffering, are likely to be much more easily visible. Thus the enthusiastic advocates of universal suffrage, of eugenics, of Bolshevism, of Americanization, are likely to speak with a zeal and passion to which the more or less academic advocate of the religion of science as the ultimate solution of all human problems is largely foreign. Dogmatism is inseparable from the minor religions, even when they are based on scientific rather than metaphysical and theological interpretations. It is perhaps most marked in the secondary religions based on theological interpretation. Most of the slaughterings and persecutions recorded in the history of Europe as being connected with the exercise of the Christian religion were not the product of contests over abstract theological dogmas or theories in themselves, but took place with reference to the application of these to very concrete problems of social adjustment, such as political and economic rights, theories

of government, or class distinctions. The bloody conflicts came usually over matters within the purview of secondary religions, seeking for a justification in the general theory of Christianity. And so it is today, except that the secondary religions now appeal more largely to scientific data and principles for their justification.

Even a great historical religion may undergo such a change as has been outlined here. The recent growth of Christianity has been from the theological and metaphysical toward the scientific explanatory system in content. It should not be forgotten that Christianity developed out of the old Hebraic religion largely as a social protest movement, or as an application of fundamental religious philosophy to some of the minor religious values of the time, such as the superiority of human functional values to formalism and ritual, the plea for economic and social justice, a struggle against an office-holding hierarchy, for an ethical as against a traditional religion, etc. The founder of the new religion lost his life in pursuing these ends. Thus Christianity was never merely theological dogma in point of content, as indeed no great religion has ever been, but more than most of the great historic religions it justified human and social values by its appeals to theological sanctions, and sometimes approached very closely to an appeal to purely human values or sanctions.

During the late Roman times and in the Middle Ages Christianity absorbed much metaphysical material and had its character largely transformed by it. The first large influx of the metaphysical, at least on the social side, was probably from contact with Stoicism and the Roman legal philosophy. But the greatest accessions of this sort were from the philosophies of Plato and Aristotle, the decrees of the various church councils and the writings and teachings of the church fathers. St. Augustine did a great deal to infuse neo-Platonism into the fabric of Christianity

on its philosophic side, thus laying a metaphysical basis for religious guidance of conduct for those who could assimilate the more abstract ideals. Throughout the Middle Ages the teachings of Aristotle often rivaled the Bible in influencing the writings and teachings of the great church philosophers, such as Saint Thomas Aquinas and Albertus Magnus. Especially on the ethical and political sides of Christianity did Aristotle's philosophic principles and his quasi-social science have great influence. The impress of the metaphyscal philosophy is also pronounced in the decrees of the church councils. Many of the more general questions settled there, as well as those disputed by the schoolmen, were primarily metaphysical in character. There were more disputes and decrees regarding essences, the nature of substances and the qualities of virtues and vices, the concepts of soul and sin, the spiritual jurisdiction of church and priest, and such matters, than there were about the nature and conduct and commandments of the gods. The mediaeval Roman Catholic church and its doctrine had become primarily metaphysical and only secondarily theological. The same fact is to be noted in connection with the Protestant Church and its discussions and decrees. Luther, Melanchton, Henry VIII, Calvin, the English theologians, the council which produced the famous thirty-nine articles, are concerned primarily with metaphysical concepts. God and the heavenly hosts are enveloped with a shroud of metaphysical verbiage and come into the clear light of day no more, except through the words of the popular preachers and the people themselves. The reason for this marked transformation of Christianity from a theological to a metaphysical religion was to be found in the fact that the prevailing methods of thinking had now become metaphysical, the theological concepts having gone into the background.

The present tendency in Christianity, at least among the leaders, is to bring it on over from the metaphysical stage to that of the dominance of the social sciences. Neither the metaphysical nor the theological elements of Christianity have disappeared from it, but probably remain more important than the scientific. This is especially true, if we consult the formal side of the religion. The official doctrines and confessions of faith, the traditional dogmas, the treatises on systematic Christian theology, are made up primarily of metaphysical and theological discussions and statements. But these elements play a constantly diminishing role in the newer Christianity. Its most vital lines of growth are in the direction of a social religious theory and a practical social service, both of which are based on the social sciences. This assertion is well attested by the fact that all the leading denominations of the Christian Church, the Central Conference of American Rabbis, and the Federated Council of the Churches of Christ, a body acting for the leading Protestant denominations have all made advanced declarations on the leading social questions of the day. These declarations embody the foremost pertinent principles of the social sciences as their primary substance and recognize the metaphysical and the theological traditions of the churches only incidentally or by way of courtesy. To be sure, these declarations have not the dignity of formal creeds, but they have much more weight with the intelligent membership than have the traditional creeds.

Also the denominations of the Christian Church have launched out on numerous social service enterprises of a great variety of types, ranging from simple programs of relief, through constructive programs of education and

social reform, to scientific investigations of social conditions as a basis for their other types of social service. These various undertakings, such as the rural survey work of the Presbyterian Church, the constructive information service of the Methodist Church, the investigational work of the Interchurch World Movement in connection with the steel industry and their program for a general national survey of social and religious conditions, are familiar to all students of social affairs. In fact, it is asserted both by friends and opponents of the tendency that the Christian Church of today is moving in the direction of becoming a great social service and instructional agency, abandoning for the most part its theological and metaphysical predilections, or at least relegating them to the category of the aesthetic and ritualistic. Certainly, through its embodiment of the principles of the social and other sciences, the church is coming under the domination of the scientific method and content. Not only does it no longer explain disease as the effect of evil spirits or earthquakes (after John Wesley) as the act of God punishing the wicked, but it expresses its religious values primarily in terms of the findings of the social sciences. The principles of sociology, psychology, economics and political science come to be embodied in the fabric of Christianity as it changes its character.

Some hold that this transformation marks the decay of religion before the growth of science. Such a view suffers from the fallacy that religion is synonomous with theology. There is no likelihood that religion will disappear, for it is the fundamental valuation process which man makes of his world and of his adjustments to it expressing the terminology and technique of thinking of the

time. In fact, many of the leading churchmen of the day firmly believe that the only way in which the great historic religions, including Christianity, can survive is to have their content and method transformed from the old theological and metaphysical concepts to the new scientific ones in which men now think. If such a transformation should not occur the historic religions might be lost, but not religion itself. In its new form it would take shelter with the various humanitarian movements and in social ethics. The traditional religions would remain, if at all, as aesthetic and cultural survivals.

The transformation of Christianity also illustrates one other tendency in religious evolution, that of the great historic religious philosophic explanatory systems to embody the approved secondary religions within their general content. At all times the newly arising secondary adjustment problems—the great problems of human origin and destiny being the primary problems—have been taken over with their accepted solutions into the implicational content, if not into the actual theory and doctrine, of the great religion of the age. Thus Christianity has always adopted or fostered certain social institutions and theories. It has, for example, made itself the champion of the monogamic family, of the state, of private property, and at times of slavery, from the time of the early church fathers to the present. Those secondary religions which are not generally accepted by the people are also rejected by the dominant historic religion, which is conservative. With the development of a more definitely scientific phase of religious content, perhaps we may expect to find all of the social programs sanctioned by the social sciences

approved by the dominant major religion, whether it be Christianity or some new synthetic religion based on science alone and neglecting Christian tradition.[6]

L. L. BERNARD.

UNIVERSITY OF MINNESOTA.

[6] It is interesting to note that this championship of minor social religions by a dominant religion may cause apparently sound social practices or theories to be repudiated by analogy or through sympathy when a protecting religion comes into disfavor. Thus the recent dislike for Christianity on the part of certain types of radicals has often been extended to include a condemnation of monogamy and much of the accepted fabric of ethics, not because social science condemned them, but because Christianity championed them and was closely identified with them.

EINSTEIN'S THEORY OF RELATIVITY CONSIDERED FROM THE EPISTEMOLOGICAL STANDPOINT

I

Concepts of Measure and Concepts of Things

THE use, which we can make in philosophy, of mathematics," Kant wrote in the year 1763 in the Preface of his *Attempt to introduce the Concept of Negative Magnitudes into Philosophy*, "consists either in the imitation of its methods or in the real application of its propositions to the objects of philosophy. It is not evident that the first has to date been of much use, however much advantage was originally promised from it. The second use, on the contrary, has been so much the more advantageous for the parts of philosophy concerned, which, by the fact that they applied the doctrines of mathematics for their purposes, have raised themselves to a height to which otherwise they could make no claim. These, however, are only doctrines belonging to the theory of nature. . . . As far as metaphysics is concerned, this science, instead of utilizing a few of the concepts or doctrines of mathematics, has rather often armed itself against them and, where it might perhaps have borrowed a sure foundation for its considerations, we see it concerned with making out of the concepts of the mathematician nothing but fine imaginings, which beyond his field have little truth in them. One can easily

decide where the advantage will fall in the conflict of two sciences, of which the one surpasses all others in certainty and clarity, the other of which, however, is only striving to attain certainty and clarity. Metaphysics seeks, *e. g.*, to discover the nature of space and the supreme ground from which its possibility can be understood. Now nothing can be more helpful for this than if one can borrow from somewhere sufficiently proved data to take as a basis for one's consideration. Geometry offers several data, which concern the most general properties of space, *e. g.*, that space does not consist of simple parts; but these are passed by and one sets his trust merely on the ambiguous consciousness of the concept, which is conceived in a wholly abstract fashion. . . . The mathematical consideration of motion in connection with knowledge of space furnishes many data to guide the metaphysical speculations of the times in the track of truth. The celebrated Euler, among others, has given some opportunity for this, but it seems more comfortable to remain with obscure abstractions, which are hard to test, than to enter into connection with a science which possesses only intelligible and obvious insights."

The essay of Euler, to which Kant here refers the metaphysician, is the former's *Réflexions sur l'espace et le temps,* which appeared in the year 1748 among the productions of the Berlin Academy of Science. This essay sets up in fact not only a program for the construction of mechanics but a general program for the epistemology of the natural sciences. It seeks to define the concept of truth of mathematical physics and contrasts it with the concept of truth of the metaphysician. Materially, however, the considerations of Euler rest entirely on the foundations on which Newton had erected the classical system of mechanics. Newton's concepts of absolute space and absolute time are here to be revealed not only as the neces-

sary fundamental concepts of mathematico-physical knowledge of nature, but as true physical realities. To deny these realities on philosophical, on general epistemological grounds means, as Euler explains, to deprive the fundamental laws of dynamics—above all the law of inertia—of any real physical significance. In such an alternative, however, the outcome cannot be questioned: the philosopher must withdraw his suspicions concerning the "possibility" of an absolute space and an absolute time as soon as the reality of both can be shown to be an immediate consequence of the validity of the fundamental laws of motion. What these laws *demand*, also "is"—and it is, it exists in the highest sense and highest degree of objectivity, which is attainable for our knowledge. For before the reality of nature as it is represented in motion and its empirical laws all logical doubt must be silent; it is the business of thought to accept the existence of motion and its fundamental rules instead of attempting to prescribe to nature itself from abstract considerations concerning what can or cannot be conceived.

This demand, however, illuminating as it appears and fruitful as the methodic stimulus of Euler proved in the development of the Kantian problem,[1] becomes problematical when considered from the standpoint of modern physics and epistemology. Kant believed that he possessed in Newton's fundamental work, in the *Philosophiae Naturalis Principia Mathematica*, a fixed code of physical "truth" and believed that he could definitively ground philosophical knowledge on the *"factum"* of mathematical natural science as he here found it; but the relation between philosophy and exact science has since changed fundamentally. Ever more clearly, ever more compellingly do we realize today that the Archimedean point on which he supported himself and from which he undertook to raise the

[1] For more detail concerning Euler and Kant's relation to him, *cf.*, *Erkenntnisproblem* (7), II, 472ff., 698, 703f.

whole system of knowledge, as if by a lever, no longer offers an unconditionally fixed foothold. The *factum* of geometry has lost its unambiguous definiteness; instead of the one geometry of Euclid, we find ourselves facing a plurality of equally justified geometrical systems, which all claim for themselves the same intellectual necessity, and which, as the example of the general theory of relativity seems to show, can rival the system of classical geometry in their applications, in their fruitfulness for physics. And the system of classical mechanics has undergone an even greater transformation, since in modern physics the "mechanical" view of the world has been more and more superseded and replaced by the electro-dynamic view. The laws, which Newton and Euler regarded as the wholly assured and impregnable possession of physical knowledge, those laws in which they believed to be defined the concept of the corporeal world, of matter and motion, in short, of nature itself, appear to us today to be only abstractions by which, at most, we can master a certain region, a definitely limited part of being, and describe it theoretically in a first approximation. And if we turn to contemporary physics with the old philosophical question as to the "essence" of space and time, we receive from it precisely the opposite answer to that which Euler gave the question a hundred and fifty years ago. Newton's concepts of absolute space and absolute time may still count many adherents among the "philosophers," but they seem definitively removed from the methodic and empirical foundations of physics. The general theory of relativity seems herein to be only the ultimate consequence of an intellectual movement, which receives its decisive motives equally from epistemological and physical considerations.

The working together of the two points of view has always come to light with special distinctness at the decisive turning points in the evolution of theoretical

physics. A glance at the history of physics shows that precisely its most weighty and fundamental achievements stand in closest connection with considerations of a general epistemological nature. Galilei's *Dialogues on the Two Systems of the World* are filled with such considerations and his Aristotelian opponents could urge against Galilei that he had devoted more years to the study of philosophy than months to the study of physics. Kepler lays the foundation for his work on the movement of Mars and for his chief work on the harmony of the world in his *Apology for Tycho*, in which he gives a complete methodological account of hypotheses and their various fundamental forms; an account by which he really created the modern concept of physical *theory* and gave it a definite concrete content. Newton also, in the midst of his considerations on the structure of the world, comes back to the most general norms of physical knowledge, to the *regulae philosophandi*. In more recent times, Helmholtz introduces his work, *Uber der Erhaltung der Kraft* (1847), with a consideration of the causal principle as the universal presupposition of all "comprehensibility of nature," and Heinrich Hertz expressly asserts in the preface of his *Prinzipien der Mechanik* (1894), that what is new in the work and what alone he values is 'the order and arrangement of the whole, thus the logical, or, if one will, the philosophical side of the subject."[2] But all these great historical examples of the real inner connection between epistemological problems and physical problems are almost outdone by the way in which this connection has been verified in the foundation of the theory of relativity. Einstein himself—especially in the transition from the special to the general theory of relativity—appeals primarily to an epistemological motive, to which he grants, along with the purely empirical and physical grounds, a decisive

[2] *Cf.* Helmholtz (29, p. 4); H. Hertz (31, p. XXVII).

significance.[3] And even the special theory of relativity is such that its advantage over other explanations, such as Lorentz's hypothesis of contraction, is based not so much on its empirical material as on its pure logical form, not so much on its physical as on its general *systematic* value.[4] In this connection the comparison holds, which Planck has drawn between the theory of relativity and the Copernican cosmological reform. The Copernican view could point, when it appeared, to no single new "fact" by which it was absolutely demanded to the exclusion of all earlier astronomical explanations, but its value and real cogency lay in the fundamental and systematic clarity, which it spread over the whole of the knowledge of nature. In the same way, the theory of relativity, taking its start in a criticism of the concept of time, extends into the field of epistomological problems not merely in its applications and consequences but even in its first beginnings. That the sciences, in particular, mathematics and the exact natural sciences, furnish the criticism of knowledge with its essential material is scarcely questioned after Kant; but here this material is offered to philosophy in a form, which, even of itself, involves a certain epistemological interpretation and treatment.

Thus, the theory of relativity, as opposed to the classical system of mechanics, offers a new scientific problem by which the critical philosophy must be tested anew. If Kant—as Hermann Cohen's works on Kant urged repeatedly and proved from all angles—intended to be the philosophical systematizer of the Newtonian natural science, is not his doctrine necessarily entangled in the fate of the Newtonian physics, and must not all changes in the latter react directly on the form of the fundamental doctrines of the critical philosophy? Or do the doctrines of the Transcendental Aesthetic offer a foundation, which is broad

[3] *Cf.* Einstein (17, p. 8).
[4] See below, Sect., II.

enough and strong enough to bear, along with the structure of the Newtonian mechanics, also that of modern physics? The future development of the criticism of knowledge will depend on the answer to these questions. If it is shown that the modern physical views of space and time lead in the end as far beyond Kant as they do beyond Newton, then the time would have come when, on the basis of Kant's presuppositions, we would have to advance beyond Kant. For the purpose of the *Critique of Pure Reason* was not to ground philosophical knowledge once for all in a fixed dogmatic system of concepts, but to open up for it the "continuous development of a science" in which there can be only relative, not absolute, stopping-points.

Epistemology, however, closely as its own fate is connected with the progress of exact science, must face the problems which are presented to it by the latter, with complete methodic independence. It stands to physics in precisely the relation, in which, according to the Kantian account, the "understanding" stands to experience and nature: it must approach nature "in order to be taught by it: but not in the character of a pupil, who agrees to everything the master likes, but as an appointed judge, who compels the witnesses to answer the questions which he himself proposes." Each answer, which physics imparts concerning the character and the peculiar nature of its fundamental concepts, assumes inevitably for epistemology the form of a question. When, for example, Einstein gives as the essential result of his theory that by it "the last remainder of physical objectivity" is taken from space and time (17, p. 13), this answer of the physicist contains for the epistemologist the precise formulation of his real problem. What are we to understand by the physical objectivity, which is here denied to the concepts of space and time? To the physicist physical objectivity may appear as

a fixed and sure starting-point and as an entirely definite standard of comparison; epistemology must ask that its meaning, that what is to be expressed by it, be exactly defined. For epistemological reflection leads us everywhere to the insight that what the various sciences call the "object" is nothing given in itself, fixed once for all, but that it is first determined by some standpoint of knowledge. According to the changes of this ideal standpoint, there arise for thought various classes and various systems of objects. It is thus always necesary to recognize, in what the individual sciences offer us as their objects and "things," the specific logical conditions on the ground of which they were established. Each science *has* its object only by the fact that it *selects* it from the uniform mass of the given by certain formal concepts, which are peculiar to it. The object of mathematics is different from that of mechanics, the object of abstract mechanics different from that of physics, *etc.*, because there are contained in all these sciences different questions of knowledge, different ways of referring the manifold to the unity of a concept and ordering and mastering the manifold by it. Thus the content of each particular field of knowledge is determined by the characteristic form of judgment and question from which knowledge proceeds. In the form of judgment and question the particular special axioms, by which the sciences are distinguished from each other, are first defined. If we attempt to gain a definite explanation of the concept of "physical objectivity" from this standpoint, we are first led to a negative feature. Whatever this objectivity may mean, in no case can it coincide with what the naïve view of the world is accustomed to regard as the reality of things, as the reality of objects of sensuous perception. For the objects, of which scientific physics treats and for which it establishes its laws, are distinguished from this reality by their general fundamental

form. That concepts, such as those of mass and force, the atom or the ether, the magnetic or electrical potential, even concepts, like those of pressure or of temperature, are no simple thing-concepts, no copies of particular contents given in perception: this scarcely needs any further explanation, after all that the epistemology of physics itself has established concerning the meaning and origin of these concepts. What we possess in them are obviously not reproductions of simple things or sensations, but theoretical assumptions and constructions, which are intended to transform the merely sensible into something measurable, and thus into an "object of physics," that is, into an object *for* physics. Planck's neat formulation of the physical criterion of objectivity, that everything *that can be measured* exists, may appear completely sufficient from the standpoint of physics; from the standpoint of epistemology, it involves the problem of discovering the fundamental conditions of this measurability and of developing them in systematic completeness. For any, even the simplest, measurement must rest on certain theoretical presuppositions on certain "principles," "hypotheses," or "axioms," which it does not take from the world of sense, but which it brings to this world as postulates of thought. In this sense, the reality of the physicist stands over against the reality of immediate perception a something through and through mediated; as a system, not of existing things or properties, but of abstract intellectual symbols, which serve to express certain relations of magnitude and measure, certain functional coördinations and dependencies of phenomena. If we start from this general insight, which within physics itself has been made very clear, especially by Duhem's analysis of the physical construction of concepts, the problem of the theory of relativity gains its full logical definiteness. That physical objectivity is denied to space and time by this theory must, as is now seen, mean

something else and something deeper than the knowledge that the two are not things in the sense of "naïve realism." For things of this sort, we must have left behind us at the threshold of exact scientific physics, in the formulation of its first judgments and propositions. The property of not being thing-concepts, but pure concepts of measurement, space and time share with all other genuine physical concepts; if, in contrast to these, space and time are also to have a special logical position, it must be shown that they are removed in the same direction as these, a step further from the ordinary thing-concepts, and that they thus represent, to a certain extent, concepts and forms of measurement of an order higher than the first order.

The fact appears even in the first considerations, from which the theory of relativity starts, that the physicist does not have only to hold in mind the measured object itself, but also always the particular conditions of measurement. The theory distinguishes between physical determinations and judgments, which result from measurement from resting and moving systems of reference, and it emphasizes the fact that before determinations, which have been gained from diverse systems of reference, can be compared with each other, a universal methodic principle of transformation and permutation must be given. To each objective measurement, there must be added a certain subjective index, which makes known its particular conditions and only when this has taken place can it be used along with others in the scientific construction of the total picture of reality, in the determination of the laws of nature, and be combined with these others into a unitary result. What is gained by this reflection on the conditions of physical measurement in a pure epistemological regard appears as soon as one remembers the conflicts, which have resulted from the lack of this reflection in the course of the history of philosophy and of exact science.

It seems almost the unavoidable fate of the scientific approach to the world that each new and fruitful concept of measurement, which it gains and establishes, should be transformed at once into a thing-concept. Ever again does it believe the truth and the meaning of the physical concepts of magnitude to be assured only when it permits certain absolute realities to correspond to them. Each creative epoch of physics discovers and formulates new characteristic measures for the totality of being and natural process, but each stands in danger of taking these preliminary and relative measures, these temporarily ultimate intellectual instruments of measurement, as definitive expressions of the ontologically real. The history of the concept of matter, of the atom, of the concepts of the ether and of energy offer the typical proof and examples of this. All materialism—and there is a materialism not only of "matter" but also of force, of energy, of the ether, *etc.*, —goes back from the standpoint of epistemology, to this one motive. The ultimate constants of physical calculation are not only taken as real, but they are ultimately raised to the rank of that which is alone real. The development of idealistic philosophy itself is not able to escape this tendency. Descartes as an idealistic mathematician was at the same time the founder of the "mechanical view of the world." Since only extension offers us exact and distinct concepts and since all clearly comprehended truth is also the truth of the *existing*, it follows, in his view, that mathematics and nature, the system of measurements and the totality of material existence, must be identified. The manner, in which the same step from the logico-mathematical to the ontological concept has been repeated in the development of modern energetics, is known. Here, after energy had been discovered as a fundamental measure, as a measure which is not limited to the phenomena of motion, but spans equally all physical fields, it was made an all-

inclusive substance, which rivalled "matter" and finally took it up into itself completely. But on the whole, we are here concerned only with a metaphysical by-way, which has not seduced science itself from its sure methodic course. For the concept of energy belongs in conception to that general direction of physical thought, which has been called the "physics of principles" in contrast to the physics of pictures and mechanical models. A "principle," however, is never directly related to things and relations of things, but is meant to establish a general rule for complex functional dependencies and their reciprocal connection. This rule proves to be the really permanent and substantial: the epistemological, as well as the physical, value of energetics is not founded on a new pictorial representation to be substituted for the old concepts of "matter" and "force" but on the gaining of *equivalence-numbers,* such as were expressly demanded and discovered by Robert Mayer as the "foundation of exact investigation of nature." (*Cf.* 52, p. 145, 237ff.)

Even in these two examples we can learn that through the whole history of physics there is a certain intellectual movement, which throughout runs parallel to the movement in epistemology that mediates and passes to and fro between the "subject" and the "object" of knowledge. Physical thought is always concerned at first with establishing a characteristic standard of measurement in an objective physical concept, in a certain natural constant. Then it is concerned, in the further development, with understanding more and more clearly the constructive element that is contained in any such original constant, and with becoming conscious of its own conditionality. For, whatever particular properties they may have, no constants are immediately given, but all must be *conceived* and *sought* before they can be found in experience. One of the most pregnant examples of this is found in the his-

tory of the concept of the atom. The atoms were postulated by Democritus as ultimate constants of nature long before thought possessed any means of concretely realizing this postulate. Fundamentally, such a realization, such a strictly quantitative meaning of the concept of the atom, was only reached in the beginnings of modern chemistry in the law of multiple proportion. To the extent, however, that to this particular realization of the concept of the atom in the law of multiple proportion others and still others are added and the concept of the atom finally comes to characterize and to organize intellectually the most diverse fields, its character as a pure principle, which was originally fused with its thing-character, comes to light. The content of the idea of the atom changes and shifts from place to place in the course of the development of physics and chemistry, but the function of the atom as the temporarily ultimate unit of measurement remains. When we pass from the consideration of "ponderable" matter to the consideration of the ether, when we seek a unity, which comprehends not only the mechanical but also the optical and electrical phenomena, the atom of matter becomes the atom of electricity, the electron. In recent physics, there appears further, with Planck's Quanta Theory the thought of an atomistic structure not only of matter but of energy. It would be in vain were one to attempt to combine all these various applications of the concept of the atom in chemistry, in the kinetic theory of gases, and in the doctrine of light and heat radiation, *etc.*, into a unitary picture. But the unity of its meaning requires no such pictorial unity; it is satisfied, indeed verified in a far stricter logical sense, when it is shown that here a common relation, a peculiar "form" of connection, prevails, which as such can be verified and represented in the most diverse contents. The atom shows itself thereby to be, not an absolute minimum of being, but a relative mini-

mum of measure. It was one of the founders of modern philosophy, Nicholas Cusanus, who, with true speculative profundity, anticipated and announced this as the function of the concept of the atom, which was to be actually realized only in the history of natural science. Cusanus' fundamental doctrine of the infinite and of the unity of opposites in the infinite rested entirely on this insight into the relativity in principle of all determinations of magnitude, on the coincidence of the "greatest" and the "smallest." (*Cf.* 7, I, 40ff, 265ff.) Modern criticism of knowledge brings the riddle, with which Cusanus' doctrine of the minimum struggles, to a simple expression. Contradiction only enters when we attempt to unify after the fashion of a thing all the different forms, which the thought of the "smallest" assumes, in the different fields of thought; but it disappears as soon as we reflect that the true unity is never to be sought in things as such, but in intellectual constructions, which we choose according to the peculiarity of the field to be measured, and which are thus in principle possessed of an unlimited variability. It follows from this that, as *what is to be measured* is unlimited in variety, so *what measures* can be represented in infinitely many and infinitely diverse ways. In other words, the unity that we have to seek lies neither in the one nor the other member, but merely in the form of their reciprocal connection, *i. e.*, in the logical conditions of the operation of measurement itself.

This receives new confirmation when we pass from the concept of matter, of energy and of the atom to the real concept of objectivity of modern physics, that of motion. The historical beginnings of the modern theory of motion in Galilei refer directly to the epistemological question, which has received its definitive formulation in the general theory of relativity. What Galilei gained with *his* idea of relativity was the cancelling of the absolute reality of

place, and this first step involved for him the most weighty logical consequences, viz., the new concept of the lawfulness of nature and the new interpretation of the particular laws of dynamics. Galilei's doctrine of motion is rooted in nothing less and nothing more than in the choice of a new standpoint from which to estimate and measure the phenomena of motion in the universe. By this choice, there was given him at once the law of inertia and in it the real foundation of the new view of nature. The ancient view saw in place a certain physical property that produced definite physical effects. The "here" and "there," the "above" and "below," were for it no mere relations; but the particular point of space was taken as an independent real, which consequently was provided with particular forces. In the striving of bodies to their "natural places," in the pressure of air and fire upwards and in the sinking of heavy masses downwards, these forces seemed given as immediate empirical realities. Only when one takes account of these fundamental features, not only of ancient astronomy and cosmology but also of ancient physics, does one understand the whole boldness of the new intellectual orientation, resulting from the Copernican system of the world. One of the most fixed and certain realities on which Grecian thought had constructed its picture of the world now became a mere illusion, a purely "subjective" feature. Even the first adherents of the new doctrine drew the decisive conclusion with reference to the doctrine of place. What Gilbert, *e. g.*, urges against the Aristotelian physics and cosmology is above all this epistemological feature, *i. e.*, that it permits the ideal and the real to flow into each other. Differences belonging merely to our thought, to our subjective reflection, are throughout made into objective oppositions. But in truth no place in itself is opposed to any other, but there are in nature only differences in the mutual positions of bodies and of material masses. "It

is not place which, in the nature of things, works and produces, which determines the rest and motion of bodies. For it is in itself neither a being nor an effective cause; rather bodies determine their mutual place and position by virtue of the forces which are immanent in them. The place is a nothing; it does not exist and exerts no force, but all natural power is contained and grounded in bodies themselves." (7, I, 360f.) It is implied in this that what we call the "true place" is never given to us as an immediate sensuous property, but must be discovered on the basis of calculation and of the "arithmetic of forces" in the universe. All determination of place—as Kepler sharply and clearly expresses this insight which for him resulted equally from astronomical convictions, physiological optics and analysis of the general problem of perception—is a work of the mind: *omnis locatio mentis est opus.* (37, II, 55, *cf.* 7, I, 339.) From this point, the way is open to Galilei's foundation of dynamics: for since place has ceased to be something real, the question as to the *ground* of the place of a body and the ground of its *persistence* in one and the same place disappears. Objective physical reality passes from *place* to *change of place*, to motion and the factors by which it is determined as a magnitude. If such a determination is to be possible in a definite way, the identity and permanence, which were hitherto ascribed to mere places, must go over to motion; motion must possess "being," that is, from the standpoint of the physicist, numerical constancy. This demand for the numerical constancy of motion itself finds its expression and its realization in the law of inertia. We recognize here again how closely, in Galilei, the mathematical motive of his thought was connected with an ontological motive, how his conception of *being* interacted with his conception of *measure*. The new measure, which is found in inertia and in the concept of uniform acceleration, in-

volves also a new determination of reality. In contrast with mere place, which is infinitely ambiguous and differs according to the choice of the system of reference, the inertial movement appears to be a truly intrinsic property of bodies, which belongs to them "in themselves" and without reference to a definite system of comparison and measurement. The velocity of a material system is more than a mere factor for calculation; it not only really belongs to the system but defines its reality since it determines its *vis viva, i. e.,* the measure of its dynamic effectiveness. In its measure of motion, in the differential quotient of the space by the time, Galilei's physics claims to have reached the kernel of all physical being, to have defined the intensive reality of motion. By this reality, the dynamic consideration is distinguished from the merely phoronomic. The concept of the "state of motion," not as a mere comparative magnitude, but as an essential element belonging to the moving system intrinsically, now becomes the real mark and characteristic of physical reality. Leibniz, too, in his foundation of dynamics, stands throughout at this standpoint, which becomes for him a starting-point for a new metaphysics of forces. Motion conceived as a mere change of place in the purely phoronomical sense, he explains, remains always something purely relative; it only becomes an expression of a true physical and metaphysical reality when we add to it an inner dynamic principle, a force conceived as an "originally implanted principle of permanence and change," *principium mutationis et perseverantiae.* (42, VI, 100 *cf.* 5, p. 290ff.) In all these examples, it is evident how sharply, on the one hand the physical thought of modern times has grasped the thought of the relativity of place and of motion, and, on the other hand, how it has shrunk back from following it to its ultimate consequences. If not only place but the velocity of a material system is to

signify a magnitude that entirely depends on the choice of the reference body and is thus infinitely variable and infinitely ambiguous, there seems no possibility of an exact determination of magnitude and thus no possibility of an exact objective determination of the state of physical reality. Pure mathematics may be constructed as the ideal doctrine of the comparison and connection of magnitudes, as a system of mere relations and functions and may come to recognize itself as such ever more clearly, but physics seems necessarily to reach an ultimate limit, a *non plus ultra*, if it is not wholly to lose any basis in reality.

The difficulty, which remains in the structure of classical mechanics in the formulation of the principles of inertia, is expressed in an epistemological circle, from which there seems no escape. To understand the meaning of the law of inertia, we need the concept of "equal times" but a practicable physical measure of equal times can, as is discovered, only be gained by presupposing, in its content and validity, the law of inertia. In fact, since Carl Neumann's work, *Uber die Prinzipien der Galilei-Newton'schen Theorie* (57), which set in motion the modern discussion on the law of inertia, it is customary in mechanics to define equal times as times within which a body left to itself traverses equal distances. Maxwell, too, in his exposition of the Newtonian mechanics, conceived the law of inertia as a pure definition of measure. The first law of Newton, as he explains clearly and pregnantly, tells under what conditions no external force is present. (51, p. 31.) Thus in the progress of mechanics the principle of inertia is *recognized* with increasing distinctness as what it *meant* fundamentally to Galilei. It is no longer taken as a direct empirical description of given processes of nature, but as the "axiom of the field," the fundamental hypothesis by which the new science of dynamics prescribes to itself a certain form of measurement. Inertia appears, not as

an absolute and inherent property of things and of bodies, but as the free establishment of a certain standard and symbol of measurement, by virtue of which we can hope to reach a systematic conception of the laws of motion. In this alone is rooted its reality, *i. e.*, its objective and physical significance. Thus, within the historical development of physics itself *what measures* is separated with increasing distinctness from *what is measured*, with which it at first seems to coincide; the observable data of experience are separated with increasing distinctness from what must be presupposed and used as a *condition* of observation and of measurement.

And what is here seen in a particular example and within a narrow field is repeated, on closer examination, in all the special fields of physics. Everywhere physical thought must determine for itself its own standards of measurement before it proceeds to observation. There must be established a certain standpoint for the comparison and correlation of magnitudes; certain constants must be established at least hypothetically and in preliminary fashion before a concrete measurement can take place. In this sense, each measurement contains a purely ideal element; it is not so much with the sensuous instruments of measurement that we measure natural processes as with our own thoughts. The instruments of measurement are, as it were, only the visible embodiments of these thoughts, for each of them involves its own theory and offers correct and useful results only in so far as this theory is assumed to be valid. (*Cf.* 8, p. 189ff.) It is not clocks and physical measuring-rods but principles and postulates that are the real instruments of measurement. For in the multiplicity and mutability of natural phenomena, thought *possesses* a relatively fixed standpoint only by *taking* it. In the choice of this standpoint, however, it is not absolutely determined by the phenomena, but the choice re-

mains its own deed for which ultimately it alone is responsible. The decision is made with reference to experience, *i. e.*, to the connection of observations according to law, but it is not prescribed in a definite way by the mere sum of observations. For these in themselves can always be expressed by a number of intellectual approaches between which a choice is possible only with reference to logical "simplicity," more exactly, to systematic unity and completeness, of scientific exposition. When thought, in accordance with its claims and demands, changes the form of the "simple" fundamental measuring relations, we stand before a new "picture" of the world with regard to content also. The previously gained relations of experience do not indeed lose their validity, but, since they are expressed in a new conceptual language, they enter into a new system of meanings. The fixed Archimedean point of the former view of the world moves; the previous ποῦ στῶ of thought appears transcended. But it is soon seen that thought, by virtue of its peculiar function, can only transcend an earlier construction by replacing it by a more general and more inclusive one; that it only shifts, among phenomena, the constancy and identity, which it cannot cease to demand, to another and deeper place. That every realization, which the demand of thought for ultimate constants can find within the empirical world is always only conditioned and relative, is guaranteed by the unconditionality and radicalism of precisely this demand. The critical theory of knowledge would not only show this connection *in abstracto*, but for it the concrete movement of thought, the continual oscillation between experience and concept, between facts and hypotheses in the history of physics, forms a perpetually new source of instruction. In the midst of the change of particular theoretical instruments of measurement, the critical theory holds fast to the thought of the unity of measurement,

which indeed signifies for it no realistic dogma but an ideal goal and a never-to-be-concluded task. Each new physical hypothesis erects, as it were, a new logical system of coördinates, to which we refer phenomena, while nevertheless the doctrine is retained as a regulative idea for investigation that all these systems converge on a certain definite limiting value. In the confusion and continuous flux of phenomena, the understanding seems at first almost arbitrarily to fix and separate out certain points in order to learn through them a definite law of change, but everything which it regards as determined and valid in this sense proves, in the course of further progress, to be a mere approximation. The first construction must be both limited and more exactly defined logically by the second, this again by the third, *etc.* Thus, ever anew does the temporarily chosen theoretical center of thought shift; but in this process, the sphere of being, the sphere of objective knowledge, is more and more penetrated by thought. As often as it seems that thought is overturned by new facts and observations, which are outside its previously formulated laws, it is seen that, in fact, thought has found in them a new point of leverage, around which moves henceforth the totality of empirically provable "facts." The epistemological exposition and evaluation of each new physical theory must always seek to indicate the ideal center and turning-point around which it causes the totality of phenomena, the real and possible observations, to revolve,—whether this point is clearly marked or whether the theory only refers to it indirectly by the intellectual tendency of all its propositions and deductions.

II

The Empirical and Conceptual Foundations of the Theory of Relativity

If there can be no doubt, according to the opening words of the *Critique of Pure Reason,* that all our knowledge begins with experience, then this holds especially when we are concerned with the origin of a physical theory. The question here can never run as to *whether* the theory has issued from experience but merely as to *how* it is based on experience and what is the relation of the diverse elements which characterize and make up the concept of experience as such. There is accordingly needed no special epistemological analysis to make clear the relation of the special and general theories of relativity to experience, to the whole of observation and of physical experiment; such an analysis will only have to decide whether the theory in its origin and development is to be taken as an example and witness of the critical or of the sensualistic concept of experience. Does "experience," as it is used here, mean merely the bare sum of particular observations—*experimentorum multorum coacervatio,* as a sensualistic thinker once described it—or is there involved in it an independent intellectual form? Is the construction of the theory merely a matter of joining "fact" to "fact," perception to perception,—or, in this connection of particulars, have there been effective all along certain universal and critical norms, certain methodic presuppositions? No "empiricism" however extreme can ever seek to deny the rôle of thought in establishing and grounding physical the-

ories, and just as little is there, on the other hand, a logical idealism, which would attempt to free "pure thought" from reference to the world of the "factual" and from being bound to it. The question dividing the two views can only be as to whether thought consists in a simple registration of facts, or whether, even in the *establishment,* in gaining and interpretation of "particular facts," thought reveals its characteristic power and function. Is its work completed in arranging particular data, immediately taken from sense perception, like pearls on a thread—or does it face them with its own original measures, as independent criteria of judgment?

The problem raised here received its first sharp and clear systematic formulation in the Platonic doctrine of ideas. For Platonic idealism, too, the proposition holds that it is not possible to think save on the basis of some perception: **οὐ δυνατὸν ἐννοεῖν ἢ ἔκ τινος αἰσθήσεως.** But the function of the "logic in us" consists indeed not in finding the sum of the particular perceptions, not in deriving and deducing the "idea of the equal" from the "equal pieces of wood and stone," but the "logic in us" is revealed in discriminating and judging what is given in perception. This discriminating constitutes the real fundamental character of thought as **διάνοια,** as *discursus.* Not all perceptions and observations stimulate equally the critical and discriminating activity of thought. There are some which do not summon the understanding to reflection since satisfaction is done them by mere sensation, but there are others which in all ways call forth thought as if in their case perception by itself could gain nothing solid. "Not exciting, namely, is that which does not pass into an opposite perception; exciting objects I call those which give opposite perceptions, because here perception gives no more vivid idea of any particular object than of its opposite. Much in perception is a paraclete of thought (**παρακλητικὰ**

τῆς διανοίας), while other perceptions are not—*such an awakener of thought, namely, is everything, which comes into sense at the same time as its opposite;* but what does not, that also does not arouse thought." (*Republic* 523-524.) In this Platonic characterization of the relation of thought and sensation, of reason and sensibility, we have, as Cohen has urged, "one of the most fundamental thoughts in the evolution of the critique of cognition." (12, p. 16ff.) Just as for Plato thought becomes what it is in assertion and contradiction, in dialectic, so not any arbitrary perception but only one to which this feature corresponds can become its awakener and paraclete. The dialectic of perception summons that of thought to judgment and decision. Where the perceptions, as it were, rest peacefully side by side, where there is no inner tension between them, thought rests also; only where they contradict each other, where they threaten to cancel each other does thought's fundamental postulate, its unconditional demand for unity stand forth and demand a transformation, a reshaping of experience itself.

The evolution of the theory of relativity has furnished a new typical proof of this general relation. It was in fact a fundamental contradiction between physical experiments from which the theory of relativity took its start. On the one side stood the investigation of Fizeau, on the other, that of Michelson, and the two seemed in their results absolutely irreconcilable. Both sought to gain an answer to the question as to how the velocity of light in a moving medium was related to its velocity in a resting medium; and they answered this question in completely opposite ways. The investigation of Fizeau showed that the velocity of light in flowing water was greater than in water at rest; that, however, not the whole velocity of the flowing water, but only a fraction of it was added to the velocity of light in a medium at rest. If we call the

velocity of light in the moving medium W and the velocity of light in a medium at rest w and the velocity of the flowing v, it results not simply that $W=w+v$, but rather that $W=w+v\left(1-\frac{1}{n^2}\right)$, in which the magnitude $n=\frac{c}{w}$ signifies the exponent of the refraction of the liquid. This result, as interpreted by the theory of Lorentz, spoke directly for the assumption of a motionless ether not carried along by the body in its movement. But the attempt of Michelson, to discover the consequences of the movement of the earth with reference to this motionless ether, failed. In no way could any influence be shown of the motion of the earth on the velocity of the propagation of light; it was rather shown with increasing evidence that all optical phenomena take place as if there were no translation of the earth against the ether.[5] And behind this conflict of "facts" there stood, as one was forced to recognize more and more, a conflict in general principles, to which the theories of mechanical and of optical and electromagnetic phenomena seemed to lead necessarily. Experiments in the latter could finally be summarized in a single proposition, the principle of the constancy of the velocity of light in a vacuum. The validity of the fundamental equations of electro-dynamics of Maxwell and Hertz involved the assumption that light in an empty space is always propagated with a definite velocity V independently of the state of motion of the body emitting it. From whatever system one made the observation and from whatever source the light issued, there would always be found the same determinate value for its velocity of propagation. But this assumption of the velocity of light as a universal constant the same for all systems, necessarily demanded by the principles of electro-dynamics, now

[5] For more detail concerning the investigations of Fizeau and Michelson as well as concerning the negative outcome of other investigations on the influence of the movement of the earth on optical and electrical phenomena, *cf.* Laue (40), p. 10ff.

comes into opposition with the principle of relativity of the Galilei-Newtonian mechanics. This principle demands that, when any definite Galileian reference body is given —*i. e.*, a body relatively to which a body "left to itself" persists in its state of rest or of uniform motion in a straight line—all the laws, which are valid relatively to this reference body K remain valid when one passes to the system of reference K′, which is, with reference to K, in uniform translatory motion. In the transition from K to K′, the equations of the "Galilei-transformation" hold,

$$x' = x - vt,\quad y' = y,\quad z' = z$$

(where v signifies a constant velocity of K′ with reference to K parallel to the x and x′ axes), to which there is to be added the identical transformation from the time $t' = t$, which is not especially noticed in classical mechanics. If we seek, however, to apply the principle of relativity of mechanics to electro-dynamics, *i. e.*, to recalculate its equations according to the formulae of the Galilei-transformation, it is seen that this cannot be done: the electro-dynamic equations, in contrast to the Newtonian equations of motion, alter their form when we insert the coördinates x′, y′, z′, t′, into them in place of the coördinates x, y, z, t according to the rules of the Galilei-transformation. The effort to unite mechanics and electro-dynamics by carrying over the principle of relativity of the first into the latter thus had to be given up: the Hertzian theory, which represented such an attempt, came into irreconcilable conflict with assured experimental results. Physical investigation stood before the dilemma of giving up a principle which had been verified without exception in all the phenomena of motion and which formed a corner-stone in the structure of classical mechanics—or of retaining it within its field but denying its applicability to optical and electromagnetic phenomena. In both cases, the unity of the explanation of nature, the unity of the very concept of nature,

seemed destroyed. Here in fact the condition set up by Plato, of the intellectual fruitfulness of experience was fulfilled: here experience stood at a point at which assured observation seemed to pass directly into its opposite. The conflict between the principle of the constancy of the propagation of light and the principle of relativity of mechanics became the "paraclete of thought"—the real awakener of the theory of relativity.

But how did physical thought go about overcoming this conflict, since it was bound to the outcome of observation as such, since it could neither set aside the facts expressed in the principle of the constant velocity of light in a vacuum, nor those expressed in the principle of relativity of mechanics? If we look back on the historical development of the theory of relativity, we recognize that the latter has followed here a counsel which was once given by Goethe. "The greatest art in theoretical and practical life," wrote Goethe to Zelter, "consists in changing the *problem* into a *postulate;* that way one succeeds." In fact, this was the course which Einstein followed in his fundamental essay, *'Zur Elektrodynamik bewegter Systeme* of the year 1905. The principle of the constancy of the velocity of light was given first place as a postulate, but,—supported by the negative result of all attempts to establish an "absolute" motion with reference to a chosen system of reference, *i. e.,* the "motionless ether,"—the *supposition* was made that there correspond to the concept of absolute rest no properties of phenomena in either mechanics or electro-dynamics, but rather that the same electro-dynamic and optical laws hold for all systems of coördinates of which the mechanical equation hold. And this *"supposition"* does not continue such, but is expressly "made a presupposition," *i. e.,* a shaping of theory is demanded which will simultaneously satisfy the conditions of the principle of relativity and those of the principle of

the constant propagation of light. (*Cf.* 16, p. 26.) The two assumptions are indeed not compatible according to the means and habits of thought at the disposal of the kinematics generally accepted before the establishment of the theory of relativity, but they—*ought* no longer to be incompatible. The demand made of physical theory was that it remove this incompatibility by subjecting precisely these means and habits of thought themselves to a critical examination. By an analysis of the physical concepts of space and time, it now appears that in fact the incompatibility of the principle of relativity with the law of the propagation of light is not to be found; that rather there is only needed a transformation of these concepts in order to reach a logically unobjectionable theory. The decisive step is taken when it is seen that the measurements, to be gained within a system by definite physical methods of measurement, by the application of fixed measuring-rods and clocks, have no "absolute" meaning fixed once for all, but that they are dependent on the state of motion of the system and must necessarily result differently according to the latter. There now arises the purely mathematical problem of discovering the laws of permutation, according to which the space-time values of an event are changed in going from one reference body to another, which is in uniform translatory motion with regard to the first. This problem is solved, as is known, by the fundamental equations of the "Lorentz-transformation":

$$x' = \frac{x - v t}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$y' = y \qquad z' = z$$

$$t' = \frac{t - \frac{v}{c^2} x}{\sqrt{1 - \frac{v^2}{c^2}}}$$

On the basis of these equations, we see that the law of the propagation of light in a vacuum is equally fulfilled for

all justified systems K and K′; on the other hand, it is seen that Maxwell's fundamental equations of electrodynamics do not change their form when the formulae of the Lorentz-transformation rather than those of the Galilei-transformation are applied to them. There is thus a universal principle of relativity, which comprehends the totality of physical phenomena; the laws, according to which the states of physical systems change, are independent of whether they are referred to one or the other of two systems of coördinates in uniform translatory motion relative to each other. (*Cf.* 16, p. 29). The principle of relativity of classical mechanics is so little contradicted by this general principle that it is rather contained in it as a special case; the equations of the Galilei-transformation directly issue from those of the Lorentz-transformation when one considers only such velocities v as are very small in comparison with the velocity of light so that the values $\frac{v}{c^2}$ $\frac{v^2}{c^2}$ can practically be left out of account. It follows from this that the principle of relativity of electrodynamics, carried over to mechanics, can come into conflict with no empirical result, while the converse carrying-over of the principle of relativity of mechanics to electrodynamics proves to be impossible, as the collapse of Hertz's theory showed. More closely considered, however, in the special theory of relativity, the electrodynamic processes are not used as a key to the mechanical, but a truly universal principle, a heuristic maxim of investigation in general, is established, which claims to contain a criterion of the validity and permissibility of all particular physical fields and theories. Thus it is seen that the initial contradiction, appearing between the principles of mechanics and those of electrodynamics, has shown the way to a far more perfect and deeper unity between them than previously existed. And this result was not reached entirely by heaping up experiments, by newly instituted investigations, but it

rests on a critical transformation of the system of fundamental physical concepts.

On the purely epistemological side, there thus appears with special distinctness in this intellectual process in which the theory of relativity originates, that peculiar "Copernican revolution," that variation in the conceptual foundations of the theory of nature, which we have previously traced in the example of classical mechanics and the older physics. An essential part of its achievement seems based on the fact, that it has shifted the previous logical constants of physical knowledge, that it has set them at another place than before. For classical mechanics, the fixed and immovable point was the assumption of the identity of the spatial and temporal values gained by measurement in the various systems. This identity was taken to be the unquestionable and sure foundation of the concept of objectivity in general: as that which first really constituted the object of "nature" as a geometrical and mechanical object and distinguished it from the changeable and relative data of sensation. **τὸ μὲν σχῆμα καθ' αὑτό ἐστι, τὸ δὲ γλυκὺ καί δλως τὸ αἰσθητὸν πρὸς ἄλλο καὶ ἐν ἄλλονς**—thus runs the proposition, which Democritus brought into the foundations of atomism, and which in modern times was taken up by Galilei to support the fundamental distinction between "primary" and "secondary" qualities, and thus the whole "mechanical" view of the world. Although the principle here established proved to be very fruitful and has been frequently confirmed in mathematical physics, the modern evolution of physics shows, with increasing evidence, that it was conceived too narrowly in a philosophical and methodological sense. The true goal of science is not mechanism but unity—as Henri Poincaré once formulated the guiding maxim of modern physics. But concerning this unity the physicist does not need to ask *whether* it is, but merely *how* it is; *i. e.*, what is the minimum of presup-

positions that are necessary and sufficient to provide an exact exposition of the totality of experience and its systematic connection. (*72*, p. 172ff.) In order to maintain this unity, which seemed endangered by the conflict of the principle of the constancy of the velocity of light and the principle of relativity of mechanics, and to ground it more deeply and securely, the theory of relativity renounces the unity of the values of spatial and temporal magnitudes in different systems. It surrenders the assumption that the temporal interval between events is a magnitude fixed once for all independently of the state of motion of the reference body and that in the same way the spatial distance between two points of a rigid body is independent of the state of motion of the reference body. By going back to the method of measuring time and to the fundamental rôle that the velocity of light plays in all our physical time measurements, it discovers the *relativity of the simultaneity* of two processes and further leads to the insight that the magnitude of the length of a body, of its volume, its form, its energy and temperature, *etc.*, are, as results from the formulae of the Lorentz-transformation, to be assumed as different according to the choice of the system of reference in which measurement takes place. But these "relativizations" are not in contradiction with the doctrine of the constancy and unity of nature; they are rather demanded and worked out in the name of this very unity. The variation of the measurements of space and time constitutes the necessary condition through which the new invariants of the theory are discovered and grounded. Such invariants are found in the equal magnitude of the velocity of light for all systems and further in a series of other magnitudes, such as the entropy of a body, its electrical charge or the mechanical equivalent of heat, which are unchanged by the Lorentz-transformation and which thus possess the same value in all justified systems of refer-

ence. But above all it is the general form of natural law which we have to recognize as the real invariant and thus as the real logical framework of nature in general. While the special theory of relativity limits itself to regarding all reference bodies K′ which are moving uniformly in a straight line relatively to a definite justified reference system K, as equivalent for the formulation of natural laws, the general theory extends this proposition to the assertion that all reference bodies KK′, whatever their state of motion may be, are to be taken as equivalent for the description of nature. (17, p. 9; 18, p. 42.) But the path by which alone this true universality of the concept of nature and of natural law, *i. e.*, a definite and objectively valid description of phenomena independent of the choice of the system of reference, is to be reached, leads, as the theory shows, necessarily through the "relativization" of the spatial and temporal magnitudes, that hold within the individual system; to take these as changeable, as transformable, means to press through to the true invariance of the genuine universal constants of nature and universal laws of nature. The postulate of the constancy of the velocity of light and the postulate of relativity show themselves thus as the two fixed points of the theory, as the resting intellectual poles around which phenomena revolve; and in this it is seen that the previous logical constants of the theory of nature, *i. e.*, the whole system of conceptual and numerical values, hitherto taken as absolutely determinate and fixed, must be set in flux in order to satisfy the new and more strict demand for unity made by physical thought.

Thus reference to experience, regard for phenomena and their unified exposition, proves to be everywhere the fundamental feature, but at the same time it is seen that,

in the words of Goethe, experience is always only half experience; for it is not the mere observational material as such, but the ideal form and the intellectual interpretation, which it is given, that is the basis of the real value of the theory of relativity and of its advantage over other types of explanation. As is known, the investigation of Michelson and Morley, which gave the impetus and starting-point for the development of the theory of relativity, was explained as early as the year 1904 by Lorentz in a manner which fulfilled all purely physical demands. The Lorentzian hypothesis, that each body moving with reference to the motionless ether with a velocity v undergoes a certain shortening in the dimension parallel to the motion, and indeed in the ratio of $1:\sqrt{1-\frac{v^2}{c^2}}$, was sufficient to give a complete explanation of all known observations. An experimental decision between Lorentz's and Einstein's theories was thus not possible; it was seen that between them there could fundamentally be no *experimentum crucis*.[6] The advocates of the new doctrine accordingly had to appeal—a strange spectacle in the history of physics—to general philosophical grounds,—to the advantages over the assumption of Lorentz which the new doctrine possessed in a systematic and epistemological respect. "A really experimental decision between the theory of Lorentz and the theory of relativity," Laue, *e. g.*, explains in his exposition of the principle of relativity in the year 1911, "is indeed not to be gained, and that the former, in spite of this, has receded into the background, is chiefly due to the fact, that, close as it comes to the theory of relativity, it still lacks the great simple universal principle, the possession of which lends the theory of relativity . . . an

[6] For more detail *cf. e. g.* Ehrenfest (15a), p. 16ff.

imposing appearance."[7] Lorentz's assumption appeared above all to be epistemologically unsatisfactory because it ascribes to a physical object, the ether, definite effects, while at the same time it results from these effects that ether can never be an object of possible observation. Minkowski too explains in his lecture on space and time that the Lorentzian hypothesis sounds extremely fantastical; for the contraction is not to be conceived as a physical consequence of the resistence of the ether but rather purely as "a gift from above," as an accompaniment of the state of motion. (47, p. 60f.) What thus, in the last analysis, decided against this assumption was not an empirical but a methodological defect. It conflicted most sharply with a general principle, to which Leibniz had appealed in his struggle against the Newtonian concepts of absolute space and time, and which he formulated as the "principle of observability" (*principe de l'observabilité.*) When Clarke, as the representative of Newton, referred to the possibility that the universe in its motion relatively to absolute space might undergo retardation or acceleration which would not be discoverable for our means of measurement, Leibniz answered that nothing fundamentally outside the sphere of observation possessed "being" in the physical sense: *quand il n'y a point de changement observable, il n'y a point de changement du tout* (5, p. 247ff). It is precisely this principle of "observability," which Einstein applied at an important and decisive place in his theory, at the transition from the special to the general theory of relativity, and which he has attempted to give a necessary

[7] 40, p. 19f.; *cf.* 41, p. 106. *Cf.* also the characteristic remark of Lorentz himself in his Haarlem lecture: "The estimation (of the fundamental concepts of Einstein's theory of relativity) *belongs to a very large extent* (*grösstenteils*) *to the theory of knowledge*, and one can leave the judgment to the latter in confidence that it will consider the questions mentioned with the necessary thoroughness. But it is certain that it will depend for a great part on the type of thought to which one is accustomed, whether one feels drawn more to the one or the other conception. As far as concerns the lecturer himself, he finds a certain satisfaction in the older conceptions, that ether possesses at least some substantiality, that space and time can be sharply separated, that one can talk of simultaneity without further specification." (46a, p. 23.)

connection with the general principle of causality. Any physical explanation of a phenomenon, he urges, is *epistemologically* satisfactory only when there enter into it no non-observable elements; for the law of causality is an assertion concerning the world of experience only when *observable facts* occur as causes and effects. (17, § 2). Here we stand before one of the fundamental intellectual motives of the theory of relativity—a motive which not only gives it the advantage over the empirically equivalent hypothesis of Lorentz, but which also produces the advance from the more limited interpretation of the postulate of relativity in the special theory to the completely universal formulation.

The way in which this advance has taken plase is especially suited to make clear the conceptual and empirical presuppositions of the theory and their reciprocal connection. The special theory of relativity rests, as has been shown, on two different assumptions, which stand, equally justified, side by side: on the postulate of the uniformity of the propagation of light in a vacuum and on the presupposition that all reference systems in rectilinear, uniform and non-rotary motion relatively to a definite justified system K are equally permissible for the formulation of the laws of nature. If one considers these presuppositions, which stand in inseparable connection in the empirical structure of the special theory of relativity, from a purely methodological standpoint, it is seen that in *this* respect they belong to different strata. On the one side, stands the assertion of a general fact, a constant of nature, which results from the experimental findings of optics and electrodynamics; on the other side stands a demand, which we make of the *form* of natural laws. In the first case, it is empirically established that there is a peculiar velocity with a definite finite value, which retains this value in any system independently of the state of motion of the latter. In

the second, a general *maxim* is established for the investigation of nature, which is to serve as a "heuristic aid in the *search* for the general laws of nature." In the formal limitation, which is placed on natural laws by this maxim, lies—as Einstein himself has urged—the characteristic "penetration" (*Spürkraft*) of the principle of relativity. (18, pp. 28, 67.) But the two principles, the "material" and the "formal" are not distinguished from each other in the shaping of the special theory of relativity. The fact that this distinction is made and that the general and "formal" principle is placed above the particular and "material" principle constitutes, from the purely epistemological standpoint, the essential step taken by the general theory of relativity. And this step seems to lead to a strange and paradoxical consequence; for the particular result is not taken up into the general, but rather is cancelled by it. From the standpoint of the general theory of relativity, the law of the constancy of the velocity of light in a vacuum no longer possesses unlimited validity. According to the general theory of relativity the velocity of light is dependent on the gravitation potential and must thus in general vary with places. The velocity of light must always depend on the coördinates when a field of gravitation is present; it is only to be regarded as constant when we have in mind regions with a constant gravitation potential. This consequence of the general theory of relativity has often been regarded as a refutation of the presupposition from which the special theory of relativity took its start and on which it based all its deductions. But with justice Einstein rejects any such conclusion. The special theory of relativity, he explains, is not rendered valueless by the fact that one comes to see that its propositions refer to a definitely limited field, namely, to the phenomena in an approximately constant field of gravitation. "Before the establishment of electrodynamics, the laws of electro-

statics were regarded as the laws of electricity in general. Today we know that electrostatics can only describe electrical fields correctly in the case, that is never exactly realized, in which the electric masses are exactly at rest relatively to each other and to the system of coördinates. Is electrostatics overthrown by Maxwell's electrodynamical equations? Not in the least! Electrostatics is contained as a limiting case in electrodynamics; the laws of the latter lead directly to those of the first for the case that the fields are temporarily unchangeable. The most beautiful fate of a physical theory is to point the way to the establishment of a more inclusive theory, in which it lives on as a limiting case." (18, p. 52.) In fact, in the advance from the special to the general theory of relativity, we have only a verification of the same principle of the construction of concepts of natural science that is found in the advance from classical mechanics to the special theory of relativity. The constants of measurement and of the theory of nature in general are shifted and magnitudes, which were regarded as absolute from the earlier standpoint, are again, with the gaining of a new theoretical unit of measurement, made into merely relative determinations valid only under definite conditions. While classical mechanics, like the special theory of relativity, distinguished between certain reference bodies relatively to which the laws of nature were valid and certain relatively to which they were not, this distinction is now cancelled. The expression of the universal physical laws is freed from any connection with a particular system of coördinates or with a certain group of such systems. To be expressed the laws of nature always require some definite system of reference; but their meaning and value is independent of the individuality of this system and remains self-identical whatever change the latter may undergo.

Only with this result do we reach the real center of the general theory of relativity. Now we know where lie its truly ultimate constants, its cardinal points, around which it causes phenomena to revolve. These constants are not to be sought in particular given things, which are selected as chosen systems of reference from all others, such systems as the sun was to Copernicus and as the fixed stars were for Galilei and Newton. No sort of things are truly invariant, but always only certain fundamental relations and functional dependencies retained in the symbolic language of our mathematics and physics, in certain equations. This result of the general theory of relativity, however, is so little a paradox from the standpoint of the criticism of knowledge, that it can rather be regarded as the natural logical conclusion of an intellectual tendency characteristic of all the philosophical and scientific thought of the modern age.[8] To the popular view and its habits of thought the radical resolution of "things" into mere relations remains as ever suspicious and alienating, for this view believes that it would lose with the thing-concept the one sure foundation of all objectivity, of all scientific truth. And thus, from this side not so much the positive as the negative aspect of the theory of relativity has been emphasized; what it destroys, not what it constructs has been comprehended. But it is remarkable to find this interpretation not only in the popular expositions of the theory of relativity but in the investigations of its general "philosophical" significance; and to meet in the latter also the view that it brings an element of subjective arbitrariness into the formulation of the laws of nature and that, along with the unity of space and time, the unity of the concept of nature is destroyed. In truth, as closer consideration shows, the theory of relativity is characterized through-

[8] Here, indeed, I can only make this assertion in a general way; for its proof I must refer to the more specific explanation in my work *Substanzbegriff und Funktionsbegriff*. (8, pp. 148-310).

out by the opposite tendency. It teaches that to attain an objective and exact expression of natural process, we cannot take without further consideration the space and time values, gained by measurement within a definite system of reference as the only and universal values, but that we must, in scientifically judging these measurements, take account of the state of motion of the system from which the measurement is made. Only when this is done can we compare measurements which have been made from different systems. Only those relations and particular magnitudes can be called truly objective which endure this critical testing, that is, which maintain themselves not only for one system but for all systems. That not only are there such relations and values, but that there must be such, in so far as a science of nature is to be possible, is precisely the doctrine the theory of relativity sets up as a postulate. If we start, as practically we must do at first, from a definite system of measurement, we must bear in mind that the empirical values, which we gain here, do not signify the final natural values but that, to become such, they must undergo an intellectual correction. What we call the system of nature only arises when we combine the measurements, which are first made from the standpoint of a particular reference body, with those made from other reference bodies, and in principle with those made from all "possible" reference bodies, and bring them ideally into a single result. How there can be found in this assertion any limitation of the "objectivity" of physical knowledge is not evident; obviously it is meant to be nothing but a definition of this very objectivity. "But it is clear," says Kant, "that we have only to do with the manifold of our presentations and that X, which corresponds to them (the object), since it is to be something distinct from all our presentations, is for us nothing; the unity, which makes the object necessary, can be nothing else than the formal

unity of consciousness in the synthesis of the manifold of presentations. Thus we say: we know the object when in the manifold of intuition we have produced synthetic unity." The object is thus not gained and known by our going from empirical determinations to what is no longer empirical, to the absolute and transcendent, but by our unifying the totality of observations and measurements given in experience into a single complete whole. The theory of relativity shows the whole complexity of this task; but it retains the postulate of the possibility of such a system all the more strenuously and points out a new way to satisfy it. Classical mechanics believed itself at the goal too soon. It clung to certain reference bodies and believed that it possessed, in connection with them, measures in some way definitive and universal, and thus absolutely "objective." For the new theory, on the contrary, true objectivity never lies in empirical determinations, but only in the manner and way, in the function, of determination itself. The space and time measurements in each particular system are relative; but the truth and universality, which can be gained nevertheless by physical knowledge, consist in the fact that all these measurements correspond mutually and are coördinated with each other according to definite rules. More than this indeed knowledge cannot achieve, but it cannot ask for more, if it understands itself. To wish to know the laws of natural processes independently of all relation to any system of reference, is an impossible and self-contradictory desire; all that can be demanded is that the content of these laws not be dependent on the individuality of the system of reference. It is precisely this independence of the accidental standpoint of the observer that we mean when we speak of the "natural" object and the "laws of nature" as determinate in themselves. Measurements in *one* system, or even in an unlimited number of "justified" systems would

in the end give only particularities, but not the true "synthetic unity" of the object. The theory of relativity teaches, first in the equations of the Lorentz-transformation and then in the more far reaching substitution formulae of the general theory, how we may go from each of these particularities to a definite whole, to a totality of invariant determinations. The anthropomorphism of the natural sensuous picture of the world, the overcoming of which is the real task of physical knowledge,[9] is here again forced a step further back. The mechanical view of the world thought to have conquered it, when it resolved all being and natural process into motion and thus put every where pure magnitudes in place of qualitative elements of sensation. But now it is seen that precisely the determination of these values, the measurements, which it applies to motions, are still bound to certain limiting presuppositions. Reflection on the manner in which we make empirical measurements of space and time shows how anthropomorphism reaches into this field that was thought withdrawn from it in principle. It is, as it were, this earthly remainder still belonging to classical mechanics with its assumption of finite fixed reference bodies and motionless inertial systems, from which the theory of relativity seeks to free itself. The conceived unit of connection determined by a system of mathematical equations here takes the place of any sensuously given, and also sensuously conditioned, unit of measurement. As is seen, there is involved here not a cancellation but a critical correction of the empirical concept of objectivity, by which a correction of our empirical spatial and temporal measures and their transformation into the *one* system of natural laws are gained.

We are brought to the same outcome by consideration of the historical problems out of which the theory of rela-

[9] *Cf.* Planck (66) p. 6ff. and (67) p. 74.

tivity has grown. To give the propositions of abstract mechanics, especially the principle of inertia a definite physical meaning had been attempted repeatedly by trying to point out some empirical systems for which they would possess strict validity. But these attempts were all thwarted, in particular, by the discovery of the motion of the solar system and of the fixed stars; to find a fixed and clear empirical meaning for the equations of the Galilei-Newtonian mechanics, nothing remained save to postulate, with Carl Neumann, an absolutely motionless body α at some unknown place in space. But such a postulate of the existence of a particular physical object, a body which can never be discovered by observation, remains the strangest anomaly, from the epistemological standpoint. (8, p. 238ff.) The absolutely motionless ether too, which seemed for a time to offer the lacking physical reference system of the Galilei-Newtonian mechanics, showed itself unsuited to this purpose; since the negative outcome of Michelson's investigation the question seemed to be decided here also. At this point, as has been seen, the theory of relativity begins. It makes a virtue out of the difficulty into which philosophical thought had fallen in its attempt to find a particular privileged system of coördinates. Experience had shown that there is no such system, and the theory, in its most general interpretation, makes it a postulate that there *cannot* and *must* not be such. That, for the physical description of the processes of nature, no particular reference body is to be privileged above any other is now made a principle. "In classical mechanics, as well as the special theory of relativity," says Einstein," a distinction is drawn between reference bodies K relatively to which the laws of nature are valid and reference bodies K′ relatively to which they are not valid. With this state of affairs no consistently thinking man can be satisfied. He asks: how is it possible that certain reference bodies (and their states

of motion) are privileged over other reference bodies (and their states of motion)? In vain, I seek in classical mechanics for something real to which I might trace the difference in the behavior of the body with reference to the systems of reference K and K'." (18, p. 49.) In this argument from the principle of *insufficient reason*, the physicist seems to move on slippery ground. One is inevitably reminded of the argument of Euler, who thought that he *proved* the principle of inertia of classical mechanics by explaining that, if a body changed its state of motion without the influence of external forces, there would be no reason why it should choose any particular change of magnitude and direction of its velocity. (23.) The circle involved here, namely, that "the state of motion" of a body is assumed to be a determinate magnitude, while it is only defined as such by the law of inertia itself, is easily seen. In Einstein's appeal to the "principle of reason," there is doubtless involved a more general and deeper epistemological motive. If we assume that the final objective determinations, which our physical knowledge can reach, *i. e.*, the laws of nature, are provable and valid only for certain chosen systems of reference, but not for others, then, since experience offers no certain criterion that we have before us such a privileged reference system, we can never reach a truly universal and determinate description of natural processes. This is only possible if some determinations can be pointed out, which are indifferent to every change in the system of reference taken as a basis. Only these relations can we call laws of nature, *i. e.*, ascribe to them objective universality, whose form is independent of the particularity of our empirical measurements of the special choice of the four variables x_1, x_2, x_3, x_4, which express the space and time parameters. In this sense, one could conceive the principle of the universal theory of relativity, that the universal laws of nature are

not changed in form by arbitrary changes of the space-time variables, as an analytic assertion; as an explanation of what is meant by a "universal" law of nature. But the demand, that there must in general be such ultimate invariants, is synthetic.

In fact, it can be shown that the general doctrine of the invariability and determinateness of certain values, which is given first place by the theory of relativity, must recur in some form in any theory of nature, because it belongs to the logical and epistemological nature of such a theory. To start from the picture of the world of general energetics—Leibniz, in establishing the law of the "conservation of *vis viva*" as a universal law of nature, referred to this logical element in it. He first defines the *vis viva* of a physical system as a quantity of work; he determines that forces are to be called equal, when they are able to perform equal mechanical work, no matter what their properties may be in detail; thus if they produce an equal degree of tension in an equal number of elastic springs, raise an equal weight to the same height, communicate to an equal number of bodies the same amount of velocity, *etc.* In this definition it is assumed that measurement of the *vis viva* by different systems of measurement will give results equivalent to each other, and thus that forces, which, when measured by a certain effect, prove to be equal or in a definite relation of greater or smaller, will retain this same relation if we measure them by any other effect. If this were not the case, Leibniz adds, and did there result a different relation of forces according to the different effect which one uses as a measure, nature would be without laws; the whole science of dynamics would be superfluous; and it would not be possible to measure forces, for forces would have become something indeterminate and contradictory, *quiddam vagum et absonum.* (42, III, 208ff.; VI, 209f.; *cf.* 5, p.

305ff.) The same process of thought has been repeated on broader physical lines in the discovery and grounding of the modern principle of energy. Here, too, the energy of a material system in a certain state was defined—*e. g.*, by W. Thompson—first as the amount of all the effects, expressed in mechanical units of work, called forth outside the system when the system passes in any way from its state into a definite but arbitrarily defined state of nullity. This explanation at first leaves it entirely undecided as to whether there exists a determinate value of what is here called "energy," *i. e.*, whether the results of the measurement of the amount of work of a system turn out the same or differently according to the method of bringing the system from the given state into a definite state of nullity. But that this determinateness in fact exists, that there always results the same amount of energy no matter what effect we use as the measure of work and what type of transition we choose, is precisely what the principle of the conservation of energy affirms. This affirms nothing else and has no other physically comprehensible meaning than that the amount of all the effects, measured in units of mechanical work, which a material system calls forth in its external environment, when it passes from a definite state in any arbitrary manner to an arbitrarily defined state of nullity, has a determinate value, and is thus independent of the type of transition. If this independence did not exist—and that it exists only experience can teach us—it would follow that what we called "energy" is not an exact physical determination; energy would not be a universal constant of measurement. We would have to seek for other empirical values to satisfy the fundamental postulate of determinateness. But it holds, conversely, that if energy is once established as a constant of measurement, it thus becomes a constant of nature also, a "concept of a definite object." Now from a physical

standpoint a "substantial" conception of energy can be carried through without arousing suspicion; energy can be regarded as a sort of "reserve supply" of the physical system, the quantity of which is completely[10] determined by the totality of the magnitudes of the states, which belong to the system involved. From the epistemological standpoint, it must be remembered that such an interpretation is nothing more than a convenient expression of the relations of measurement, that alone are known, an expression which adds to them nothing essential. The unity and determinateness of measurement can be immediately understood and expressed as the unity and determinateness of the object, precisely because the empirical object means nothing but a totality of relations according to law. It follows from this analogy from a new angle that the advance in "relativization" which takes place in the theory of relativity, represents no contrast to the general task of objectification, but rather signifies one step in it, since, by the nature of physical thought, all its knowledge of objects *can* consist in nothing save knowledge of objective relations. "Whatever we may know of matter," here, too, we can cite the *Critique of Pure Reason,* "is nothing but relations, some of which are independent and permanent and by which a certain object is given us." (34, p. 341; *cf.* Müller's Trans. p. 232.) The general theory of relativity has shifted these "independent and permanent relations" to another place by breaking up both the concept of matter of classical mechanics and the concept of the ether of electrodynamics; but it has not contested them as such, but has rather most explicitly affirmed them in its own invariants, which are independent of every change in the system of reference. The criticism made by the theory of relativity of the physical concepts of objects springs thus from the same method of scientific thought,

[10] In more detail in Planck (63) p. 92ff.

which led to the establishment of these concepts, and only carries this method a step further by freeing it still more from the presuppositions of the naïvely sensuous and "substantialistic" view of the world. To grasp this state of affairs in its full import we must go back to the general epistemological questions offered to us by the theory of relativity; we must go back to the transformation of the *physical concept of truth* involved in it by which it comes into direct contact with the fundamental problem of logic.

ERNST CASSIRER.*

UNIVERSITY OF HAMBURG.

* This article is a selection from a translation of Professor Ernst Cassirer's *Zur Einstein'schen Relativitätstheorie*. A translation of Professor Cassirer's *Substanzbegriff und Funktionsbegriff*, with the whole of the former essay as a supplement, will be published in the course of the year by THE OPEN COURT PUBLISHING COMPANY as one volume under the title *Substance and Function*. The translators are W. Curtis Swabey, Ph.D., and Marie Collins Swabey, Ph.D.

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CRITICISMS AND DISCUSSIONS

SCIENCE AND SELF-SACRIFICE

THIS generation has seen illustrated in full a paradox more startling than those of Kant. We have seen thesis and antithesis spread upon the front pages of our newspapers, and the whole world is working out the solution. The thesis was put in what seemed unassailable terms in the political campaign cries of a dozen years ago. "The full dinner pail" and "dollar diplomacy," the free silver agitation, free trade and protection, each rested its case on a foundation which only a few visionaries questioned, and to them no one paid heed. This foundation was the belief that self-interest was the dominant factor in life. That the individual's will to live, to enjoy life and play a large part in it, to conquer foes and live on the spoils of the victory, was the fundamental motive in all action. The Christian teaching of the value and necessity of self-sacrifice played no part in practical politics or in business life. The world believed in morality, but it very distinctly disclaimed any belief in Christian morality. Justice meant something, a man had a right to his own, but if he had nothing, it was no one's duty to sacrifice *themselves* to give him of their plenty. In broad outline this was the active and real creed of the world up to 1914. Where exceptions are to be noted, especially in the foreign policy of the United States toward Cuba and toward China, they were looked upon as the magnanimous throwing to weaker folk of the crumbs spilled from our overflowing table. A man or a nation had first to look out for itself, then and only then would other things be added to *it*.

No one needs to be reminded of the change today. Four years have worked a revolution. There are men still living in the past, aging politicians who cannot read the signs of the times. But the tide of popular demands rolls on, and a socialist government in once imperial Germany is called moderate, and our prayers go up for its continuance, lest something more radical threaten us. To wage war, politicians and statesmen had to appeal to something other than self-interest. To give up money, ease, comforts, and life itself was but a glorious opportunity to fulfil the highest demands of life. The completeness of this change we have not yet realized. Self-sacrifice is taken as a matter of course, and if my brother has

less than I, it is my duty to feed him, even though I take from the scanty store of my own table. Self-sacrifice and service are the foundation of the appeals to which the world is responding.

The self-sacrifice given by millions has won the great struggle, yet the war is not formally ended nor decently interred before self-interest, and conflicting claims as to mutual rights and interests are again heard. The swing of the pendulum was so rapid that there is question whether it may not swing back as rapidly. Thesis and antithesis are before us in plain sight. The problems of the solution have come upon us before we were ready.

In the eagerness to down the foe, men forgot for a moment their doubts, but now those doubts return. Can men be depended upon to give themselves to service and to self-sacrifice when the war passion has passed? Was it not perhaps the Berserker fury of war, after all, and not the gentle pleading of Jesus of Nazareth that brought about this outpouring of self? If man continued to act on the basis of the Sermon on the Mount, would it pay? Is there any real place in the world for the Christian doctrine of self-sacrifice? Does a man benefit when he does right? Are those forces in the world which make for righteousness powerful enough to overcome the operation of the natural laws of self-interest and the struggle for the survival of the fittest? Does a moral Being who rewards righteousness and punishes iniquity rule the material universe? Or does blind force determine the destinies of men? Is the end of all the struggle and sacrifice to be that some time the last man, though he be benefited by all that the generations have done for him, must perish weakly in some natural upheaval? Is all that we have gained in the years of war at the mercy of some wandering star that may any moment end all life upon the earth? These old questions return again to men's minds.

These doubts return, but one even more fundamental comes with them. There has been noted in this war the same tendency to fatalism on the part of the soldiers that has appeared in wars of the past. Not what a man does, but what fate has in store for him, determines his safety or death. A man should strive to the utmost, but that striving will not assure safety. This is perhaps inevitable under circumstances where doing one's duty so frequently leads to greater and greater danger. The will to serve faithfully is the will that frequently leads to death. Were the soldier not so faithful he might perhaps escape. He has no thought of this, but leaves to fate the result. For the soldier faithful service assures no material

reward. Under the impulse of the combat, with the war spirit strong, this brings no questioning, but when the ardor of conflict dies, the question recurs with redoubled force. Of what use is faithfullness since it gives no material reward? The soldier does what he must. Of what avail for him is the struggle to be faithful overmuch?

This is but the restatement of the problem of the place in the world of the will to do good. A statement by Professor H. C. Warren of Princeton (*Journal of Philosophy*, August 15, 1918, p. 464), puts this problem distinctly before us. "All human activity, including deliberation and selective volition, is completely mechanistic." If all life is fundamentally mechanistic, functioning as a supermachine, then it is not a righteous Providence watching to reward faithful service that rules the universe, but a blind medley of forces, which may at any moment be completely altered in character by the coming of a stronger force. The arrow sent by chance killed King Ahab, not any retribution at the hand of a righteous God. The connection between Ahab and the arrow is mechanical, not moral. So, fundamentally, is every connection of life if this theory be true. We need then to face squarely the issue. If all life is mechanistic, explainable and governed by unmoral forces, then self-sacrifice, the devotion of the soldier as well as of the saint, is quixotic and useless.

There is a plain opposition between mechanical force as a theory of existence and a moral deity. Yet the solution is not as simple, nor the differences as easy to state as *it* seems at first. Facts are facts, and form the material with which either theory of life must build. Moreover, either theory must use all the blocks in the puzzle. In the final building all the facts must have their appointed place. Mechanism must account for morality as a fact in the world, and theism must account for the death of King Ahab by a chance shot. We turn first to the facts of morality.

Evolution did not cease when man reached his present physical form. More change has *occurred* in his methods of obtaining a living, more changes on the surface of the earth due to modern civilization than in any previous period in the world's history of many times the duration of our modern world. The evolution is now an evolution of ideas and collective ability. An united family can do more than a single individual. More game falls to the hunter hunting with his brothers than when alone. The clan and tribe *is* more efficient than the single family in providing for human needs. A nation brings greater security and material prosperity to its citizens

than independent and warring tribes. It is at least a present dream that an united world will be more efficient and prosperous than separate hostile nations.

The one thing that has made these larger social groups possible is morality. As men's ideas of duty toward their fellow men *has* grown and broadened, so the social unit has enlarged. As men have learned to sacrifice and serve each his brother, evolution has moved forward. Social morality has made possible life in the family, in tribes, in great cities, in nations and in empires. Without the teaching of duty to one's neighbor man would be even more solitary than packs of dogs in their hunting. The dogs find it possible to work together without sacrificing each his own private interests. This man cannot do. Men must either fight or serve. For savage tribes a stranger is an enemy. Only when bound by ties which impose mutual obligation can another man be trusted.

This social obligation has been inforced by the tribal and national religions. One's duty to one's brother is strengthened by one's duty to one's tribe. Because he, too, is a member of the tribe I must aid him. Because he, too, worships the tribal god I must give to fill his need. As tribal religions make possible the tribe, so the Christian religion makes possible a world brotherhood.

Morality, and the religion that reinforces morality, *has* played a large part in the progress of the human race. They are factors that must be included in any study of existence. If the world is mechanistic, the mechanistic theory must explain morality and religion, and account for their work as evolutionary factors.

As the fact of morality must be accounted for, so must its dependence on the material world. It must be brought into relationship with that world. Just how is the chance death related to the intentional infliction of the death penalty? Just how far does the field of morality extend? Morality has had to do not only with intra- and inter-tribal life, it has also almost as often, in its primitive stages, concerned itself with the relation of the savage to the animals and plants around him. He has at times sought to force the rain to fall, and to this end has enforced rigorous prohibitions upon the members of the tribe, and especially upon the priests or leaders. If some modern views of morality have divorced it from the material world and ignored that world, the savage never fell into that error. He built up a tribal morality, enforced certain ceremonies and required certain forms of self-denial in an effort to gain power over animal and inanimate nature. As he sought to

force other tribes of men to his will, so he sought to rule the wild animals of the forests, and even the rain and the stars. He never sought, as we do, to build up a morality unconcerned with the material needs of life. It is to a later stage that the statement belongs that the rain falls on the just and on the unjust like. For the savage, only to those who live the proper life and perform the proper ceremonies will the spirits of the rain be gracious. That the rain does not come in response to the charms of the medicine man we know, but that knowledge has not stopped man's effort to rule nature.

If we were content to rest in modern ethical theories, it would be enough to prove the existence of morality as a necessary factor in *human* life. The problem would then be for the mechanist to build his theory in accordance with this fact. But we cannot rest content with this. If only while one lives has morality any place, then the fields of Europe mark the downfall of all ethical theories. If all that man dies for is that his fellowman may live and in his turn die, we cannot escape the question, of what avail is it? If there is nothing in the best human character that is not always at the mercy of a chance arrow, if the whole life of a city which men have stricken to make pure, for which they have toiled and suffered, may in an hour be utterly destroyed by a volcanic outburst, and only the criminal in jail escape, our ethical and religious theories fall. Better for Red Guards to take and enjoy while they may, than to suffer to no avail. Religion must face this problem squarely, if religion is to continue in the world.

The case is not as dark for religion and morality as our statement might indicate, nor need man take refuge just yet in mysticism and declare physical life an illusion and not to be taken into account. The savage took it into account, and the modern man takes it into account. Modern morality is not confined to making men live together as brothers. We combine not merely in order to defend and protect ourselves against our fellow man, but also to gain power over the material world. A race that spans sea and land with railroad and telegraph, that sends boats beneath the waves, and men to fly thousands of feet in the air, that predicts the weather, even though it cannot control it, that provides as a matter of course against the cold of winter and the heat of summer, has gone far in the conquest of nature. Complete conquest is not achieved, but every year sees some step forward in this mastery of the universe. All this would be impossible without modern morality. Only by those combinations which the sense of moral obligation makes pos-

sible has man increased or held his advantage over inanimate life. Religion, in supporting and strengthening morality, is the driving force in this conquest of the world. Theories of religion must take account of this fact. The savage failed in his attempt to rule all existence only because he did not have the proper weapons. The same morality which he used, only differently applied, the same sense of mutual obligation, is the foundation of our successes in the conquest of the world.

The fact that men from before the dawn of history up to the present have set themselves to this conquest of the world, and to that have subordinated everything, ease, wealth, even the individual life, forces us to change our question. It was the passion of war that made men forget self and made them willing to die for an idea. It was the passion of the fighter that has led to the voluntary suffering of every reformer. It is in the heat of the conflict that through all the ages the hero has shed his blood. When the conflict has come, when love of ease and self has been opposed by love of *tribe, and the* will to war against an enemy of that tribe, whether that enemy be man or matter, only the coward and craven has sought to save self. In war, self-sacrifice is more fundamental than self-seeking. On this rock *every form* of utilitarianism falls in pieces. The call to stand, even in death, side by side with one's brother awakes a stronger response than the desire for personal safety. It is more primitive. It comes more directly from the womb of nature. Morality apart from this cannot exist. And religion and morality are bound indissolubly together. It is self-seeking and not self-sacrifice that needs to be explained. As in this war just over, so in all conflicts of the past, the burden of proof rested upon the man who declined to sacrifice self or possessions. The man who gave of self or of money gave as a matter of course, and without question. Our problem must then be restated. We do not need to account for self-sacrifice; but we have still to deal with the modern question as to its wisdom.

The fact that self-sacrifice is primitive and instinctive goes a good way toward proving it of value. Just as the instincts of the lower animals are developed in order better to equip the animal in the struggle for existence and the better to fit him to his environment, so human fundamental traits play the same part. We have at times looked too high for our justification of morality. We have valued the human race so little that only those things are called of value which aid in the everlasting preservation of the race. To

maintain the race for a day, if tomorrow the race is destroyed, we hold of no account. Nature does not so count value. Whatever even merely tends to preserve a race, though it may not assure its safety, is of value. Man is in the world. If he is to continue in the world a certain amount of co-operation between man and man is necessary. Morality assures this co-operation. Therefore, morality is of value in the material world.

The continued existence of the race demands self-sacrifice of the individual. This he renders, in a sense from necessity, in a sense also, of his own free will. No man acts solely after an exhaustive study of the facts and of the probable results of a contemplated action. No man lives to himself alone. The old idea of individualism, as well as of the value of the individual soul, is not Christian, and is the result of non-Christian influences. The insistence of Jesus that he who wills to save his soul shall lose it, and that he who loses his soul shall save it, should have warned us that our current theology was in error. Nowhere does Jesus teach that a man apart from his fellows is of infinite value. "The whole creation groaneth and travailleth together until now." It is neither instinctive nor Christian for a man to demand personal advantage to himself from every action he takes. Thus the Christian does not demand an answer to the question, whether sacrifice of self is of benefit to the individual. It is enough for him that the race, his brothers, benefit by his toil and suffering.

The man who is not a Christian, who asserts the right of the individual to demand personal benefit from every personal endeavor, still insists upon his question, of what advantage to me is self-sacrifice. When the call for service comes, a man finds himself bound to his fellowmen by many ties. The sinking of the Lusitania, the sufferings of the Belgians and Armenians, pull at his emotions. He can refuse to heed the appeal, but in so doing he breaks those ties which were a fundamental part of his nature. He does violence to his own truest self. This is not religious cant, but a statement of biologic and natural law. The man who refuses to sacrifice self for the benefit of his brothers is no longer the man he was or might be. He has cut off a part of himself. Material safety he may find, but at personal loss. His future acts, assisting only in the well being of one only among the millions of creatures on the earth, cannot have the value that self-sacrifice would have had. It is the refusal to serve that is without value in this material world of ours.

A man must serve, must sacrifice self, or be no true man. Yet, for all this his labor, for all the benefit that may come to his brothers from his toil, there may be in the end no result. If the race is of no value to the universe, if man makes but a brief and impotent appearance upon the stage of life, then man's fate is tragic. Serve and sacrifice self he must, or be untrue to his fundamental instincts, yet when he is the nearest to his ideal, a few ages may sweep away all that he achieved. If the end of the world be a cold lifeless planet, then morality has no final place in time. Then those forces in the world which make for morality are not fundamental in the universe. Man for his own peace of mind may live gloriously as a hero while he lives, but like the Scandinavian gods, for all that the heroes may do, their Asgard is bound to go down to destruction. Whether man's lot is thus tragic is the real problem.

To answer such a question is beyond our present purpose. Yet we need not therefore sit down beside the rivers of Babylon and weep. What the future may hold in store for the world we do not and cannot know. It is not, however, a matter to be decided by the natural sciences alone. It is a fact in the material world, that the human race exists. The material forces have given a place to man. In a sense morality and religion are natural phenomena, concerned with the existence of a part of the material world. Whatever the nature of the fundamental forces of the universe, those forces cannot be described in terms of geology and astronomy alone. Those forces are the forces which include spiritual forces. On the other hand spiritual forces concern and imply material forces. The Logos Doctrine has at least this analogy to modern scientific theories, that it tends to portray God as being forced by his own nature to express Himself, and that expression includes expression in material form. This is the assumption of all theories of the Incarnation. Neither for Christian theology nor for science can we divorce morality and religion from the material realm. As part of that realm morality has value. Even if the world become lifeless, that lifeless husk will bear traces of man. It will not be the same word it would have been without man. What the value of a lifeless trace of an extinct race may be, is not our present question. Man has changed the world which he found. Morality has made possible those changes. Therefore, morality has a value for the material universe. Men through the influence of morality and religion have played an *irradicable* part in existence.

More than this. As science must explain how from lifeless matter came living organisms, so for the future it must explain how those spiritual forces can be gathered back again into a lifeless void. In neither case can we rest content with the present scientific theories. As no mechanistic theory can be suffered to call itself truly scientific if it ignores the place of morality and of religion as we have sketched it, so no astronomical theory can be allowed to assert unchallenged that the spiritual forces are at the mercy of the material forces. No man can look far into the future. The only sure foundation for prophecy is the look forward we obtain from a tower built on the foundation and erected from all the facts of life. When the astronomer takes into account the spiritual forces, then, and not until then, need religion concern itself with his prophecies. So long as he omits religion and morality and organic life from his calculations, he cannot be sure that his conclusions are true. The neglected factors may upset all his carefully calculated results.

The moral forces are forces active in the natural world. Yet they are not chemical or mechanical. We do not need to delve into the structure of being to maintain this contention. Nor are we just at this moment arguing against the mechanistic constitution of matter. Moral and religious influences can neither be described, correlated, nor explained, in the terms of a chemical or mechanical science. History and ethics do not use the terms nor the methods of chemistry and mechanics. Whatever may be their constitution, whether the result of chemical reactions or not, the moral forces as we see them active in the world are of a different type from the inorganic reactions. We can explain a man sitting in the sun on a cool day as we explain the tropisms of the unicellular organisms, much as we explain the affinity of certain substances for one another, but we have no chemical terms that will account for the elaborate heating systems of modern days. This is not due merely to the ignorance of the chemist. Scientific theory to be efficient must account for details. The chemical history of the plans and purposes involved in the building of a house would be so complex and involved that it would be useless as a working theory. The simple statement of the purpose of the builder is effective as the chemical account would not be. Chemical and mechanical theories of life, therefore, do not apply to morality and religion, and the forces that bring about a moral life—that is, the spiritual forces—have a right to treat the world from this moral point of view. These moral forces are what the religious man calls the power of God. We have, there-

fore, a right to include God among the natural forces. In any complete survey of existence He must be taken into account. In this sense, but in this sense only, we have proved His existence.

Just so far as man has dominion, so far has man's god dominion. Whatever triumphs lie before man, his god will lead him to them and share in them. So long as men live in communities on the earth and their combined endeavors rule inanimate nature, so long will the god who rules them exercise his power over the universe. All this, however, does not answer the question raised by the prophecy of a dead world where man does not exist. Nor does it answer the questions raised by the escape of the criminal and the death of all the righteous in the volcanic eruption. This latter point, however, is easily set aside. It is easily conceivable that in the end man may succeed in harnessing the volcanoes, and other natural forces. In this present struggle to master them, no one claims that he has won the final victory. Each defeat but spurs him to greater exertion and some day we may expect to see the result. For this it is worth while to maintain and increase the efficiency of the moral forces which make it possible for men to work together. A momentary defeat is no disproof of the value of morality.

The first objection still stands. If the fate of the earth is to become a dead world, cold and still, or to end in some explosion of superheated gasses, man cannot hope finally to rule all existence. Even today he cannot influence the stars. There is the possibility which some extreme visionaries dream of, the ability of man to pass through interstellar space before the end of life on this planet and to transfer to another world, Venus or some more distant star, the life that has begun here. The bare possibility of this the astronomer cannot deny, but this can bring little solace to the moralist. To pass interstellar space would not be possible to even a completely moral world unless there *shall* by that time be discovered some physical means to pass beyond our atmosphere and still live. The continued existence of man would then depend not on morality but on physical science. Not a moral God, but an all-knowing God would or could bring this about. In discussing such a dream we are no longer in the sphere of morality. We may therefore acknowledge our limitations and leave the question. If a man waits to be moral until he is assured that the results of his moral action will assure human life into endless ages, he will never act at all.

There is another side even to science. Natural forces are to be understood when most fully revealed. The forces which brought

this earth into being are most clearly revealed in earth's highest product, the human race. To the human race we must turn in order to understand what the natural forces really are. There is a scientific theory that every physical event leaves some trace, has some effect on the universe. As we have already noted, man and morality must therefore have an everlasting influence, even though man himself cease to exist. To this corresponds the religious conception of the eternal value of the soul.

Religion has two aspects. It is both social and mystic. Men have sought to divorce the two, but always in every religion that has endured, the two are to be found inextricably intermixed. We have been speaking of morality as simply *a* natural *phenomena.* It is more than this even for science. Morality is a matter of man's consciousness. It is therefore a proper study of the science of psychology. It is still more. In acting from a sense of duty man is self-conscious. Personality comes into play. It is no longer possible to exhaust the problems of the moral will by a purely objective study. We, therefore, pass over to the world of introspection.

When a man is moral, when he resists some suggestion on the ground that it is not "right" for him so to do, he is conscious of his own will. The same is true on the positive side. When a man enlists in the army because he thinks that he ought to, he places side by side the idea of country and his duty to his country. It is himself that he studies and is conscious of, his own will and that focus of activities we call the self. This is true of any deliberated action. For religion it is also true. In fact we have the claim of the higher religions of the right to exercise dominion over the whole will of man. Each and every action is either right or wrong. The questions which we raised in the beginning have no place here. A man has no right to require proof that the act will benefit him, before he acts. The categorical imperative is an element in all religions. Man must obey first and reason afterward. Social pressure is to the same end. Conform to custom first and attack it afterward, but conform. Safety for the human soul consists in this conformity. By conforming, the soul attains immortality, whatever immortality may be. Right action, action that expresses the will to do right, is of value altogether apart from results. I am to love my brother, to give him alms, present day Moslem and *Medeaeval* monk agree, not so much to help him as to store up virtue for myself. I am to be moral not to save others but myself. Not to save the race of men, but to do God's will is the aim of religion. Whether the race

continues to exist or not is of little value. In fact religion has at times taught that very little of the race will be saved. The world is due for destruction. Only those who are to be saved out of the world are to be considered. The scientific problems of the future of this earth are thus ignored.

We have to face this attitude clearly. Though often crudely put, yet in its essence it is scientific. As a matter of fact man does get satisfaction in expressing himself in action regardless of the result. "After me the deluge" expresses much of human conduct. So long as my will has expression, so long as I do what I intend, I may not care whether the effects last long or not. Only when my purpose concerns all men yet to be born do I need to raise the question as to the future of the human race. The statesman who builds for all time must face the question whether it is worth while to labor for a passing and temporary result. The man of religion who is content to save one soul from the burning may well pass lightly over the problem. We have to ask whether this religious attitude has any possible scientific justification. Has morality any place in reality apart from its results on the material universe?

A will or purpose is a conscious state which seeks to perpetuate itself. If the purpose relates to the outer world, then the perpetuation sought is in the material universe. If the purpose concerns the individual consciousness alone, then the end sought is the continuance of that conscious state. When the end sought is found, the man is far from content. Other purposes may come, but that purpose or desire is expressed in the result attained. The picture the artist paints, so far as he attains to his ideal, is the expression of the beauty he desires to portray. The moral will share this characteristic. The moral act, as the expression of the will to do right, is the fulfilment of that purpose. The moral act expresses the intention to be moral. Since morality is the expression of the effort of the human race to continue to exist and to expand, the moral act is the expression of traits fundamental in humanity. It could not be the human race and not labor to build up co-operative effort. It could not form great kingdoms, empires, and republics except by the expression of itself in moral action. By moral action we mean to include self-sacrifice. Self-sacrifice, fundamental and primitive, expresses one of the primitive and essential instincts in man. Whatever the results of moral action, it has at least this value always, that it gives expression to this fundamental human instinct. It expresses the power of the race over the individual, and gives con-

sciousness to the race soul. In this is a fact which must form the basis of any scientific study of man.

It is here that morality is tied up indissolubly with religion. Morality has this value in itself only as the expression of human instincts and forces which seek expression. As morality concerns itself with the actions resulting, so religion deals directly with those forces. All that religion does come back in one way or the other to the effort to build up a moral dynamic. Religion thus has an inescapable value to the human race. A desire becomes a purpose and hardens into will only as it attains a certain measure of expression. The willing of an action means the accomplishing of that action so far as the individual can bring it about. Religion strengthens those desires and brings them more readily to expression. Selfsacrifice is instinctive, but the constant insistance by the Christian teachers on its necessity undoubtedly brings mankind more readily to this act. The moral strength of the allied powers finally overmatched the limits of self-sacrifice of the Germans largely because the religious element was more clearly present with the allies. Defeat did not daunt them as it did the Germans, because they were more prepared for self-denial. Self-sacrifice as a religious duty thus has its value apart from its results. Because it gives expression to man's true self, it is of value in itself.

The moral will, and especially the will to deny one's self, not merely expresses a fundamental instinct, it is conscious of that self-fulfillment. This is the mystical element in religion. Even the mystic has often misunderstood it, and tried to divorce his will to deny himself from the race for which that denial is to be made. The Buddha tried this, but even he stayed in the world to teach his brothers the truth. Compassion for them affected even the Enlightened One. The saint who denies himself for his brother's sake is fulfilling the highest function of a member of the human race. He is doing that for which he came into the world. He is expressing his manhood.

Man is an expression of the meaning of existence. The universe may have other more complete expressions of the real significance of its life, but human kind is at least a partial expression. No other race or existence known to us reveals as much of what life means as does man. He is for us the highest point attained by the cosmic powers. Those instincts which express most truly his nature therefore reveal best the meaning of the universe. Self-denial is fundamental to man, and expresses his truest nature. Self-denial

is therefore a true expression of the meaning of life. Those forces which reveal themselves in morality and religion are revealing the true nature of the universe. Thus, again, religion and morality have a value apart from any temporary or incidental results. We do not ignore the problem as to the relation of man to the material universe. Religion and science must face these problems, but no solution of those problems can take from the value we have found for self-denial. The Christian doctrine shows itself to be scientific and true to the facts. One question we can answer finally. Since morality and religion have and represent a truer conception of the facts than is possible to material science, no conclusions of mechanics or chemistry can overthrow our results. They may explain life for their own purposes in mechanical terms but the true explanation of the facts of human life rests with the moralist and the saint.

Not merely is the true nature of man revealed in the will to serve, but in addition man is conscious of his will. He knows what this true self is, even more surely than he knows it to be his true self. We have been arguing all this time as to whether morality is fundamental in human nature, but no argument is needed to tell man what morality is. He knows by direct experience what his will and spirit *is*. So far as the true being of the universe is revealed in man, in knowing himself he also knows that true being. In man's will to serve he therefore comes into direct contact with this true being. This true being is what he calls God. In willing to sacrifice himself for his fellows, and in being conscious of that will to serve, in recognizing that it is his true self, man also recognizes that he touches directly the divine spirit. Such language seems to affront both science and religion. Such affront appears only because even today science and religion are too far sundered. If God is real, he must influence and be the true being of the material universe. As such he concerns both science and religion. He who truly knows God knows the truth of all existence, and he who knows the real being and nature of the world knows God.

In this way, mysticism, the conscious presence of the divine spirit in man, attains a valid place in scientific study. We need to notice that it is a mysticism that reveals the world, and not a mysticism that withdraws from the world. Only to the man who wills to serve is it possible to become conscious of God's presence. Ancient and classic mysticism failed because in the effort to gain self-consciousness there was left out everything of which one could be conscious. Yet the search was partially successful. Men found by

searching within something which they recognized as their true being. If we, in the fullness of our understanding of the necessity of service, come to self-consciousness, we will not fail as they failed in making that communion with God really conscious and distinct.

We have not been directly discussing the question whether God exists. If we define God as the true being, or as the truth of existence, then by definition He exists. He may not be personal, or infinite, or endowed with power, but whatever life is, that is God. Our problem in such a case is to find out what this truth of existence is. In conscious devotion and self-sacrifice man finds contact with this true being. So, in the religious phrase, he finds God. He finds Him to be a conscious power. Meaning is given to man's life by this consciousness of service. As this will to serve reveals man's true nature, so it reveals God, who is the true being of the universe. As man's will is a conscious power, so God is known as a conscious power. What more He may be, whether consciousness is the highest form of life, needs no decision here. As man is the highest form of life known to us, so the revelation of *God* in man is the highest revelation possible to us. God is at least all that man knows him to be.

The paradox that we stated at the beginning is thus brought to a certain kind of solution. The purpose to provide for one's self and conquer the world is primitive and fundamental in man, but so also is the instinct to conquer by combining with one's fellows. For this self-sacrifice is essential. The solution is that the instinct for self-preservation is really the effort to express the true nature of our existence. This, however, is better expressed by the spirit of co-operation, for it is co-operation which raises man above the brute. Self-expression rather than self-preservation is the real fundamental instinct. Sometimes it shows itself in the effort of the individual to gain for himself such power that none may oppose his will. Sometimes it shows itself in the self-sacrifice of the individual as he thereby expresses his purpose of joining with his fellows in a common warfare. This is the synthesis by which the opposing ideals of the last generation and of ours are gathered to one common end.

This synthesis or solution is purely in scientific terms. We have asked neither the metaphysical nor the religious value and meaning of our problem. We have put aside the questions as to the final outcome of man's effort to subdue nature. We have been dealing with facts and not values. As a matter of fact we found that self-sacrifice, along with self-assertion, expresses man's true being,

and in so far as man is part of the universe, expresses partially the true being of our universe. In this sense we have found out something about the nature and being of God. For all this we do not need ourselves to be religious. Only if we are not religious we must take our facts at second hand. To the teacher and leader in religion, such a study as this must be made and the results taken into account if he is to make the right connection between religious and secular life. The pendulum will of necessity swing back again perhaps to its old extreme of each man for himself unless it is stopped or slowed up in its swing.

For theology, the study of the meaning of God's life and of man's relation to Him, such a conclusion as ours must be taken into account. A metaphysics can come to conclusions concerning the meaning and relation of what is already known to us. It cannot bring us new facts. Theology can relate the facts known to us, as a philosophy of the Christian religion, but so long as it remains a philosophy, an interpreter merely, it can give us no new facts. To prove God's existence we need facts. If already in fact we have known God, then philosophy or theology can explain these facts. Prove it, however, it cannot. Meaning is not fact. If God is a being who is a fact in existence, then He will be known as a fact and not as an inference. What He is will be known as a fact, and that knowledge be subjected to scientific study. As "fact" we have found God and learned something of His nature.

Yet, once more, we have been studying facts, not explaining them. How it comes to pass and why, that man is called to self-sacrifice which may destroy him in spite of his instinct to live, how and why it is that man is the highest point so far attained in the evolution of the world; what the end and the value of human life may be; upon all these questions we gain no light from our present study. Whether that spirit which we have met as the true spirit of man is destined to rule all existence, whether our God is omnipotent and eternal, we know no more than before. All this remains for future study. We rest content at this point with having found man's religious nature to be fundamental to his existence.

GEORGE A. BARROW.

CHELSEA, MASS.

DEATH

By D. T. Praigg

Oh, Death! thou long maligned and dreaded foe
Of that inherent spark which comes unsought,
But unto which we cling as though it were
Of priceless worth, I come to thee with meed
Of praise too long delayed, for thou hast been,
Of all the friends of man, the truest, best,
And steadfast most in loyalty and love;
And hast evinced from him a sympathy
Which thy detractors, lost in blind conceit,
And dreading change from fickle light of Day
To cloudless Night, can ne'er appreciate,
Nor plaudit give for duty well performed.

Thou dost come to man when others from him
Turn away, and he becomes an outcast
On the paths of earth, shunned, reviled, abused
By all his fellows, and afflicted sore
By heavy hand which Time upon him lays;
And thou dost give to him the gentle boon
Of rare forgetfulness of worldly griefs,
Op'ning wide to him the regal chamber
Of honored guest, where blest Oblivion
Close draws the curtains of her silence oe'r
The din of conflict in a world of strife,
And gives to tired life her sweet repose.

Thou art the loyal friend, oft tried, of strength,
The enemy of weakness, self-approved,
Yet thou dost come unto the old, infirm,
And long despairing with reward of rest,
And lead them far away from earthly paths
On which they tread with falt'ring step and are
A burden to themselves and earth and time;
And for this sad estate dost give to them
Thy blest eternity of voiceless Calm.
And taking thus the debris from the paths,
Which Time doth litter with the wrecks of men,
Thou givest Youth an unobstructed course
On missions that unveil the New and make
The roads of earthly progress bud and bloom
With fragrance and with beauty unexcelled.

Thine is the heritage of blissful Calm,
In which the Present reigns supreme, undimmed
By clouds that lower o'er the buried Past
And from misgivings of the Future free,
In an abode where Hope's illusions cease
To beckon on to dull Despair, and Time,
Of pow'r despoiled, incites no more to aim
Whose inspiration is its earthly tomb.
Thus, then, to pay thee tribute and declare
The regal worth of thy decree, I come,
And on thy paths I strew the bloom of earth
And crown thee Mercy's noblest gift to man.

THE ECONOMICS OF MORALITY

THE national welfare may be called the national economy, since in the national welfare a plurality of interests becomes a community, or economy, of interests. The national welfare is the one interest that virtually represents all of the interests of the nation; therefore, it is the most important interest of the nation. And every other interest of the nation is half dependent on this leading interest, just as the final, or real, effectiveness of every individual workman is half dependent on the officials, the leaders, of the business in which he is engaged.

Now, as an interest, the national welfare is dealt with in two very different, though complementary, ways. The first way is by means of codes of political laws. It is extremely difficult for many, if any, men to grasp and retain in mind the end, or ideal, of national welfare in its relation to present conditions; and therefore, these codes of political laws are substituted in men's minds for the real end of national welfare. It would be out of the question to decree that all men act in whatever way gives best promise of helping to attain the end of national welfare, for since most men's minds are very confused in regard to the proper method of attaining this ideal, there would result confused and conflicting actions and modes of life. It would be impossible to establish the guilt of offenders. But codes of political laws are easy to understand, and therefore to obey. They deal with particular modes of action, rather than action in general. They make clearer to men what their duties are with respect to political, or civil life, and in so doing they enable courts to punish offenders more readily.

Codes of laws are based solidly on the experience of the race in trying to attain the end of national welfare. Whatever modes of action have proven useful in attaining the end of national welfare are made legal; whatever modes of action have proved injurious are made illegal. Now it sometimes happens that the experience on which a law is based is forgotten by the nation, and that the nation fails to enforce the law, or changes it, until experience again proves

its wisdom. But as a general rule laws are enforced, even if their *raison d'etre* be temporarily forgotten. Authorities realize that it is very hard for most men to realize the importance of many laws, or to foresee the full consequences of disobeying any law, and therefore authorities enforce laws with a strictness corresponding to the importance of the laws. It is demanded of men that they obey laws faithfully and somewhat blindly.

It is very necessary that men establish codes of laws to assist in attaining the end of national welfare; but codes of laws are not all that men need. They are intended chiefly for men's *wills*, since they must be obeyed rather blindly, for men's intellects cannot have anything like free play in respect of laws. Yet the intellect is just as important as is the will, and provision should be made for the intellect. The will should be obedient, and the intellect clear.

To satisfy this need men have developed the science of political economy. Principles of political economy are not enforced, as are political laws; for things that are left to the intellect are to be decided by the intellect. Principles of political economy are based solidly on the experience of the race, as are political laws; but political laws are *commands*, whereas principles of political economy are *assertions*, or *suggestions*. Political laws compel the individual, principles of political economy assist the individual. People *must* have political laws, since there can be no government without laws; and people *should* have principles of political economy, since governments cannot be efficient where political economy is unknown. Between the right-of-ways of laws there is much individual freedom of action; and in order to make the most of this freedom of action, individuals are taught political economy, as far as is practicable. It is not necessary for all persons to understand political economy—only a small part of the population of any nation understands it—yet the welfare of all nations is at least double what it would be if political economy were unknown.

Now *morality* is also an economy; it is the *universal* economy of interests. Morality is the one interest that represents all other interests; it unites all interests. In morality all other interests have their being. All parties to life are irrevocably committed to morality; it is the chief end of life. Whenever another interest becomes exaggerated, conflicts with morality, it must sooner or later give in. Its very life depends on morality, as a branch depends on its vine. In attaining the moral end it is important to use two different, though complementary methods analogous to those used in attaining the

end of national welfare. In the experience of the race certain modes of action have proved beneficial; these have become morally legal. Other modes have proved injurious; these have become morally illegal. It is hard for most men to foresee the consequences of disobeying these moral laws, and therefore they are expected to be obeyed almost blindly as well as faithfully, like political laws.

With the universal welfare, morality, as with the national welfare, it is important that people develop and teach principles of economy—in this case principles of *moral* economy. Men should be obedient to moral laws, and should understand principles of moral economy. Between the right-of-ways of moral laws there is always room for a very great amount of individual freedom of action. Therefore, the individual will always be able to do more good if he understands principles of moral economy. And just as the national welfare *is now* twice what it *would be* if political economy were not taught, so it is that morality, the universal welfare, *would be* twice what it *now is* if moral economy were taught.

It cannot be denied that morality is more than a question of economy; for morality cannot be adequately defined in terms of any interest, even as virtue, happiness, wealth, honor, or economy, since these are all elements in all interests. Morality is like a cord of many threads. All of the threads lead to the end of the cord, to the moral end; and therefore any thread will serve as a guide. Because, to be truly moral an act must satisfy all of the threads of cord; it must satisfy at once the demands of virtue, happiness, honor, wealth, economy. If it fails to satisfy all of these it will be found to fail to satisfy any. And if an act fully satisfies any one of these it will be found to satisfy all of these. If one had exhaustive knowledge of the nature of any one side of morality it would suffice to guide one to right action in all things, without knowledge of the nature of any other side of morality. But it is impossible to have such knowledge. Therefore, one should not attempt to live up to the requirements of only one side of morality, to regard morality as a question of only one interest; for just as an object is seen better in two lights than in one, so morality is understood better in two lights than in one. Morality is without doubt just as much a question of economy as it is a question of virtue, happiness, wealth, or honor. And in connection with this it may be said that the national welfare is a question, not only of economy, but also of virtue, happiness, wealth and honor, since all of these are inseparably connected with the national welfare. The difference between the

national welfare and morality, the universal welfare, is therefore mainly a difference in degree; the national welfare is a limited interest, whereas morality is an unlimited interest.

Just as morality is the representative of all other interests, and therefore the equal in importance to all other interests combined, so moral economy is the representative of the economies of all other interests combined. Moral economy is the economy of the whole body of interests; and the economy of *any whole* is exactly equal in importance to the combined economies of *all parts.*

It may be thought by some that teaching principles of moral economy means the interference on the part of some men in the internal affairs of others, but such is not the case. For teaching general principles is part of the work in all sciences, yet no one regards teaching other sciences as an interference on the part of some men in the internal affairs of others. For the individual is almost wholly dependent on general principles in dealing with his particular problems. The individual will always have his individual problems, and no science attempts to solve them for him. Principles of moral economy, like principles of political economy, seem clearer as they are studied in relation to one another.

The following are fundamental principles of moral economy, and give an idea of the general nature of the subject:

1. The more leading one's position is in any field, the greater is his chance of seeing the needs of that field in their true relation.

The officials of any business see the needs of that business more clearly than do the employees of that business; the leaders of an army see the needs of that army, the materials and movements necessary, better than do men in the ranks. It is the same with all governments, whether city, state, or national; it is the same with all interests *in* the world, and with the general interest *of* the world. There is much unrest at present, and therefore people do not readily see the truth of this principle. Now it is true that there are some cases in which leaders do not see the needs in their field so clearly as do some of their followers, but the cases are relatively very few. The exceptions cannot disprove the rule; with this, as with a mosaic, the parts do not give a perfect idea of the whole. It is easier to see the truth of this principle in dealing with classes than it is in dealing with individuals; for there can be no doubt but that the leading classes in all fields see the needs of their fields more clearly than do other classes, and as we come down the scale of classes we find that

the lower the class in any field, the less clearly it sees the needs of its field.

II. The more urgent any need in any field becomes, the greater becomes its chance of receiving the attention of leaders in its field.

This is moral economy's law of supply and demand. It is based on Principle I. All true needs are *moral demands;* and whenever any need becomes more urgent, whenever there is a greater moral demand for anything, the need receives greater attention from leaders in its field. All needs, all interests are related, and if any need becomes sufficiently urgent it will not only receive attention from the leaders in its immediate field, but it will also receive attention from the leaders in related, often more important, fields. There are, of course, exceptions to this principle. However, there are fewer real exceptions to it than there are apparent ones. For instance, most people have exaggerated ideas of the importance of some needs; and the leaders in the fields of these needs, who see the real importance of these needs, and who give them the proper share of their attention, seem to be neglecting these needs. This seeming neglect causes people to think that leaders do not see needs in their proper relations to one another, or that, if they do, they do not attend to needs according to their urgency. In this, people are assuming as a basis of argument the very thing that they undertake to prove. Of course, leaders, like their followers, have a limited amount of attention to divide among needs; they must be frugal with their attention. Though leaders work to the limit of their capacity, they cannot give any need all of the attention it deserves. They, therefore, endeavor to find out which needs deserve their attention most, and which needs may best be left to their subordinates. This principle does not mean that leaders should be left *alone* to attend to needs; for no leaders can accomplish much more than other men, unless their followers do their share of the work. Followers must co-operate with leaders, everyone must make as great an effort as possible; but since followers often interfere with leaders by trying to advise them, or by acting with too little confidence in them, it is important that followers keep in mind the nature of leadership. This principle should be remembered in choosing work, whether it be work of a few hours, or life work. For it is seldom that people do not overvalue some lines of work. It is quite the usual thing for people to think that certain lines of work are more important, and more neglected than they are, unless this law of supply and demand is considered. There are everywhere misfits, who

know they are misfits, yet who continue their present work, thinking it to be important, and neglected. In a few instances they are right, but in most instances they are wrong. If they all understood this principle, most of them would change to other occupations which are more useful, and at which they have more ability.

Now a philanthropist may understand scientific philanthropy, and yet not know how to do most actual good with his money. For scientific philanthropy is not so comprehensive as is moral economy; a gift may seem of greatest value from the standpoint of scientific philanthropy, and yet, from the more comprehensive standpoint of moral economy the gift may seem of value, but not of greatest value. For moral economy is concerned in all needs, no matter what their nature; it overlooks none.

III. The greater the difference in position in any field between two men in that field, the less the chance of its paying society for one of them to attempt to advise or consult the other about matters concerning that field.

There is little that ordinary men in any field know that men of importance, leaders, in that field do not also know; and therefore men should be careful in advising men of more importance than themselves in any field about matters concerning that field. Also, men should be certain that their need for advice is sufficiently urgent before they ask men of more importance than themselves for advice; for advising requires time, and the more important the man, the more important his time. Men of more importance should also remember this—they should be certain that they can get good advice from the man of lower position before they ask him for advice, and they should be certain that the man of lower position is sufficiently in need of advice before they take time from their important work to advise him. Of course, large bodies of men, through their spokesmen, are more likely able to give useful advice than single men, or small bodies of men; and large bodies of men are more likely to be worth advising than single men, or small bodies of men. Three things, therefore, should be considered in advising, or consulting; the *urgency* of the need for advice, or the *certainty* that one's advice will prove helpful; the *size* of the body of men that needs advice, or wishes to advise; and the *importance* of the man or group, of men that needs advice, or wishes to advise.

IV. In choosing one's work, whether it be work of a few hours,

or life work, one should give equal consideration to two things: the usefulness of the work; and one's ability to do the work.

Many people choose their work according to their ability to do it; many others choose their work according to the usefulness of the work. Everyone should give equal consideration to both factors, and choose the work that, from both factors, promises to be the most useful in which he can engage. This work will in most instances be neither the work for which he has the very greatest ability, nor that which is most useful, or important. Because, one usually has *much greater ability* for some *much less useful* lines of work, and also *much less ability* for *much more useful* lines of work. One may have very little ability as a statesman, and very great ability as a janitor, and decide to be neither, to be an engineer, or journalist. In estimating the usefulness of any line of work one should not forget Principle II, for it is impossible to make anything like a correct estimate, unless moral economy's law of supply and demand is considered.

On these fundamental principles of moral economy it is possible to build a system quite as vast as that of political economy, and in all probability much more so, since moral economy is the representative of all other economies, since moral economy is fully comprehensive.

Now morality seldom receives the credit or attention it deserves: there is a vague feeling among some men that, even were there no moral order, things would somehow, in some order, function, and perhaps, in some material respects, grow or develop. Few see that morality is the sole interest that, like gravitation, at once holds things together, and properly separates them. As was shown above, the efficiency of all bodies of men or interests is dependent half on the leaders of the bodies. The vast majority of men have a very indefinite idea of the proportion of welfare, or efficiency, that is due their leaders. Almost all admit that the leaders are the most important men of any body of men with respect to the efficiency of that body, but few men admit that half of the efficiency of most bodies of men is due to the leaders. As a general thing, leaders individually receive *in amount more*, and *in proportion to the value of their services less*, than do followers. The individual usually overrates his own importance, and underrates that of his leaders; he usually over-rates the interest, or economy, in which he is interested, and underrates the leading interest of all—morality—and the leading economy of all—moral economy. This is simply part of the universal tendency

to bring things to a general level in respect of rewards, or credit for, services rendered. Few men will agree that moral economy deserves a place in education, and in thought, ahead of political economy; but to one who has considered all sides of moral organization this conclusion is unavoidable.

In the field of every economy there are possible lines of action ranging all the way from those whose economic value is clear to those whose economic value is obscure. All in all, common sense is less than half so adequate as it is if combined with knowledge of principles of economy. Principles of moral economy should not overrule moral laws, but should be applied where they do not conflict with moral laws. Moral laws should receive all of the attention that they now receive, and probably more; but principles of moral economy should receive, in relation to moral laws, just as much attention as do principles of political economy, in relation to political laws.

There is already education in morals to quite an extent; but there is no education in moral economy whatsoever. The science of moral economy is very important, very urgent; the development of this science will yield greatest returns for the effort spent on it. Scientific philanthropy, social efficiency, political engineering, and kindred subjects, are somewhat like moral economy; but all of these are relatively narrow in scope, they might be called *branches* of moral economy. Principles of moral economy are universal principles; they are useful to all men who are trying to do good, useful in all walks of life.

B. H. SOMERVILLE.

NORWOOD, VIRGINIA.